Modeling and fault detection of UAVs in closed-loop setups

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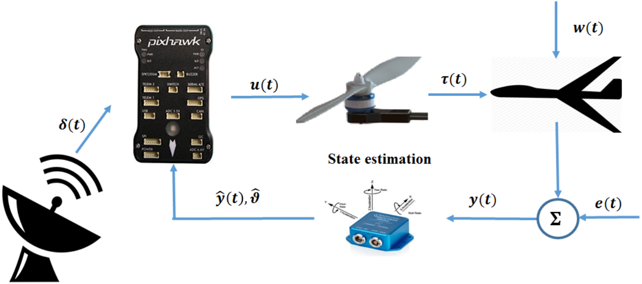
Motivation

Goal: Modeling and parameter estimation for an UAV with extension to fault detection.

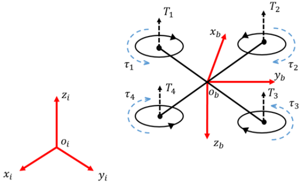


- Challenges: closed loop setup, cheap sensors and unknown parameters.
- Approaches: consistent estimations of forward model and inverse models.

Modeling of a quadcopter

The translational dynamic model of a quadcopter is

$$m\dot{\boldsymbol{V}}_b + \boldsymbol{\nu} \times (m\boldsymbol{V}_b) = m\boldsymbol{R}^T\boldsymbol{g} + \boldsymbol{T}_z - \boldsymbol{\lambda}\boldsymbol{V}_b$$



Symbol	Quantity
m	Mass of the quadcopter [kg]
$\dot{\boldsymbol{V}}_b$	Accelerations of the quadcopter [m/s^2]
$\boldsymbol{\nu} \times (m\boldsymbol{V}_b)$	The centrifugal force [N]
$\boldsymbol{g}$	The gravity vector [m/s^2]
$\boldsymbol{R}$	The rotation matrix
$\boldsymbol{T}_z$	The total thrust [N]
$\boldsymbol{\lambda}$	The drag coefficient matrix [Ns/m]

Lateral model for a quadcopter

- Lateral dynamic model:

$$\dot{v} = g \cos\theta \sin\phi - \frac{\lambda_1}{m} v + \bar{\tau}$$

- IMU sensor measurements:

$$\dot{\phi}_s = \dot{\phi} + e_{\dot{\phi}}$$
$$a_{y,s} = g\cos(\theta)\sin(\phi) - \dot{v} + e_y = \frac{\lambda_1}{m} v + e_y$$

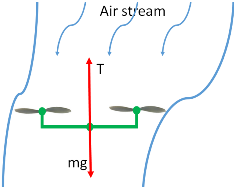
- Indirect model (that describes the dynamic relation between the sensor signals):

$$a_{y,s} = \frac{\frac{\lambda_1}{m} g}{\mathsf{p}(\mathsf{p} + \frac{\lambda_1}{m})} \dot{\phi}_s + e$$

Vertical thrust of the quadcopter

Refined thrust modelling

$$T_z = \bar{c}_1 \sum_{i=1}^{4} \omega_i^2 + \bar{c}_2 \sum_{i=1}^{4} \omega_i = k_1 \underbrace{\sum_{i=1}^{4} u_{\text{ci}}^2}_{u_{in1}} + k_2 \underbrace{\sum_{i=1}^{4} u_{\text{ci}}}_{u_{in2}}$$



Lateral dynamic model:

$$\dot{w} = -\frac{T_z}{m} - \frac{k_w}{m} w + g \cos\theta \cos\phi$$

IMU sensor measurements:

$$a_z = \frac{T_z}{m} + \frac{k_w}{m} w + e_{a_z}$$

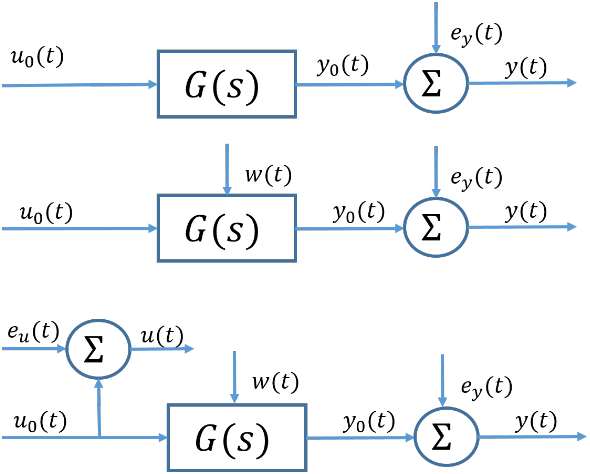
Transfer function model:

$$a_z = \frac{\mathsf{p}}{\mathsf{p} + \frac{k_w}{m}} \left(\frac{k_1}{m} u_{in1} + \frac{k_2}{m} u_{in2} \right) + \frac{\frac{k_w g}{m}}{\mathsf{p} + \frac{k_w}{m}} + e_{a_z}$$

and the standard thrust model is $k_2 = 0$.

Estimation problem?

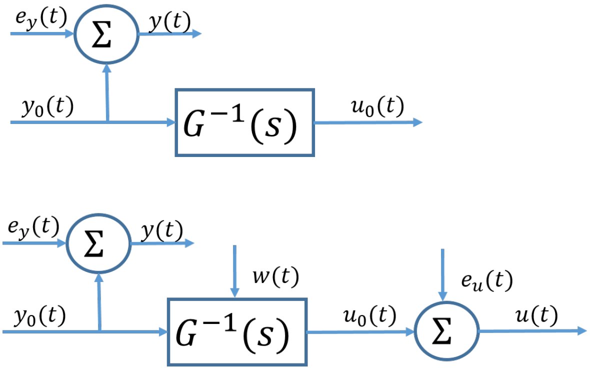
Is it possible to achieve consistent estimates of the parameters regarding three models?



One solution is to use Instrumental variable estimation method.

Second problem?

Is it possible to consider the inverse estimation scenarios?



Do we gain or lost anything regarding the inverse model estimations?
And if e_y and e_u are not independent,the problem will be error-in-variable and closed loop.

Estimation of forward and inverse models

Consider a simple model

$$y(t) = \frac{0.01q^{-1}}{1-0.995q^{-1}}u(t) + \frac{1}{1-0.995q^{-1}}e(t) = G_F(q)u(t) + H(q)e(t) \tag{1}$$

where $e(t) \sim \mathcal{N}(0,\, 0.85^2)$ and $u_0(t) = \frac{1}{1-1.95q^{-1}+0.975q^{-2}}\delta(t)$,
$\delta(t) \sim \mathcal{N}(0,\, 1.0^2)$.
The inverse model is

$$u(t-1) = 100y(t) - 99.5y(t-1) - e(t) = G_I(q)y(t) + T(q)e(t) \tag{2}$$

Using Least square estimator for Monte Carlo simulation of 1000 runs, yields

Forward model	Inverse model
$-0.9948 \pm 0.0014,\ (-0.995)$	$11.0717 \pm 10.9000,\ (100)$
$0.0100 \pm 0.0004,\ (0.01)$	$-10.9528 \pm 11.2787,\ (-99.5)$

The bias estimate of the inverse model is due to the correlation between $y(t)$ and $e(t)$, which implies the error-in-variable is more complicated.

Using instrumental variable

The EIV (or closed loop) and inverse model estimation problem could be handled using IV method. The result is submitted to in the 18th IFAC Symposium on System Identification, SYSID 2018
Du Ho and Martin Enqvist, On the equivalence of forward and inverse IV estimators with application to quadcopter modeling

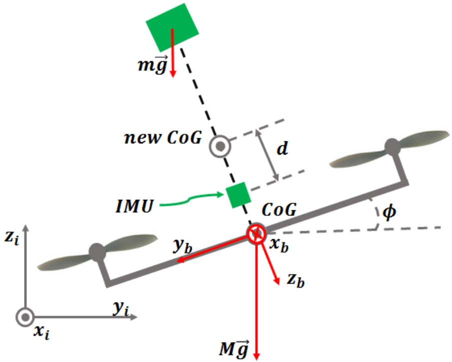
The estimates of the parameters of the forward and inverse models obtained from the simulation study with $u(t) = u_0(t) + e_u(t)$ when the basic IV method is used.

True parameter	Forward model	Inverse model
-0.995	-0.9947 ± 0.0018	-0.9947 ± 0.0018
0.01	0.0100 ± 0.0005	0.0100 ± 0.0005

The differences of the parameter estimates obtained from the simulation study with $u(t) = u_0(t) + e_u(t)$: $\hat{\theta}_F - \hat{\theta}_I$ and $\text{std}(\theta_F) - \text{std}(\theta_I)$

True parameter	Mean estimate	Standard deviation
-0.995	0.6178×10^{-7}	0.2504×10^{-7}
0.01	-0.1812×10^{-7}	0.0217×10^{-7}

Application to quadcopter's center of gravity estimation



The translational model is

$$\dot{v} = g \sin\phi - \frac{\lambda}{\tilde{m}} v,$$

The IMU measurements are

$$\dot{\phi}_m = \dot{\phi} + e_{\dot{\phi}}, \quad a_y = g\sin\phi - \dot{v} + d\ddot{\phi} + e_{a_y}.$$

Forward model and inverse model

The model from the lateral acceleration measurement to the measured roll rate is

$$\dot{\phi}_m = \frac{p^2 + \frac{\lambda}{\tilde{m}} p}{dp^3 + d\frac{\lambda}{\tilde{m}} p^2 + g\frac{\lambda}{\tilde{m}}} a_y + e_F = G_F(p) a_y + e_F.$$

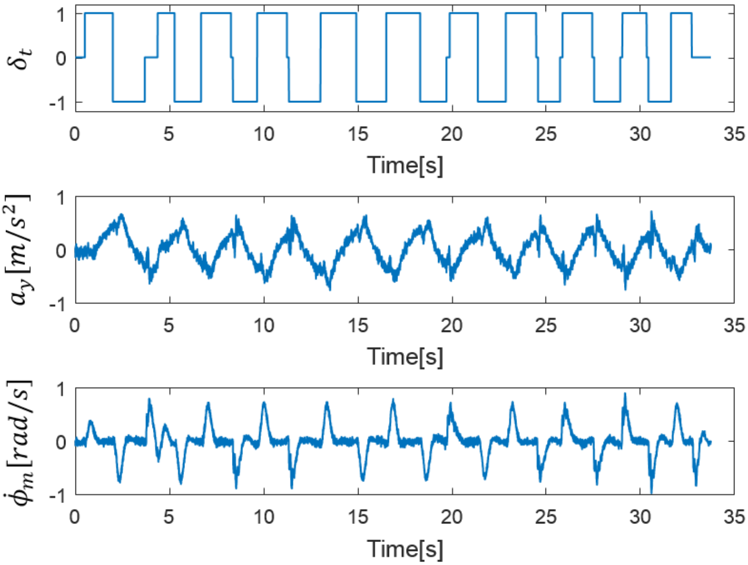
Discretizing the forward model using $p = \frac{q-1}{T}$ which gives

$$G_F(q) = \frac{\alpha_1 q^{-1} + \alpha_2 q^{-2} + \alpha_3 q^{-3}}{1 + \beta_1 q^{-1} + \beta_2 q^{-2} + \beta_3 q^{-3}},$$

and the inverse model will be

$$a_y = \frac{\frac{1}{\alpha_1} + \frac{\beta_1}{\alpha_1} q^{-1} + \frac{\beta_2}{\alpha_1} q^{-2} + \frac{\beta_3}{\alpha_1} q^{-3}}{1 + \frac{\alpha_2}{\alpha_1} q^{-2} + \frac{\alpha_3}{\alpha_1} q^{-3}} q\dot{\phi}_m + e_I.$$

Experimental result



Parameter estimation

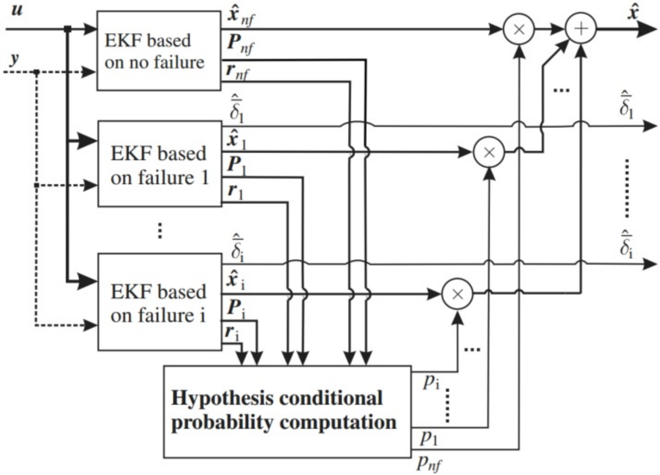
Par	Forward ($\hat{\theta}_F$)	Inverse ($\hat{\theta}_I$)	std	Inverse ($\hat{\gamma}_I$)
α_1	0.1103	0.1104	0.1696×10^{-3}	$\widehat{\frac{1}{\alpha_1}} = 9.0546$
α_2	-0.2204	-0.2206	0.3349×10^{-3}	$\widehat{\frac{\alpha_2}{\alpha_1}} = -1.9975$
α_3	0.1101	0.1102	0.1657×10^{-3}	$\widehat{\frac{\alpha_3}{\alpha_1}} = 0.9975$
β_1	-2.9917	-2.9917	0.1739×10^{-3}	$\widehat{\frac{\beta_1}{\alpha_1}} = -27.0882$
β_2	2.9834	2.9834	0.3426×10^{-3}	$\widehat{\frac{\beta_2}{\alpha_1}} = 27.0132$
β_3	-0.9917	-0.9917	0.1691×10^{-3}	$\widehat{\frac{\beta_3}{\alpha_1}} = -8.9794$

Then The estimates of the CoG d with standard deviations are obtained from the forward model and the inverse model, respectively.

Forward model ($\hat{d}_F$)	Inverse model ($\hat{d}_I$)
4.5312 ± 0.00696 cm	4.5273 ± 0.00695 cm

Fault detection and isolation

Extended adaptive multi model estimation



Modeling

The model using in each EKF is modified

$$y_{mf}(t) = y_m(t) + \sigma(\bar{y}(t) - y_m(t))$$

$$x_a^n = \begin{bmatrix} x \\ \bar{y}(t) \end{bmatrix}$$

where σ equals 1 if a fault happens in this sensor, otherwise σ equals 0. The $\bar{y}(t)$ is fault value which can be modeled as a random walk $\bar{y}(t+1) = \bar{y}(t) + w_{\bar{y}}(t)$ with $w_{\bar{y}(t)}$ to be modeled as a white noise.
For each EKF, the probability is computed as

$$p[r_i(k)] = \frac{1}{(2\pi)^{\frac{n_y}{2}} |S_i(k)|^{\frac{1}{2}}} e^{-\frac{1}{2} r_i(k)^T S_i(k)^{-1} r_i(k)}$$

The goal: Investigate the possibilities to use different measurements as the inputs/outputs.

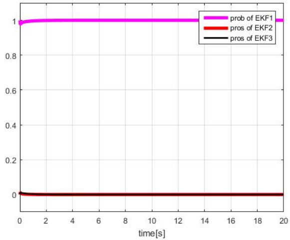
Example

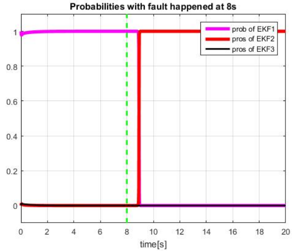
The pitch dynamic model is formulated as

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha & Z_q \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} Z_{\delta_e} \\ M_{\delta_e} \end{bmatrix} \delta_e + w(t)$$

$$\begin{bmatrix} \alpha_m(t) \\ q_m(t) \end{bmatrix} = \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix} + e(t)$$

The coefficient values using for simulation are $Z_\alpha = -0.6, Z_q = 0.95, Z_{\delta_e} = -0.115, M_\alpha = -4.3, M_q = -1.2, M_{\delta_e} = -5.157.$
Hypothesis test result





Summary and future work

- Summary
  - The inverse model estimation is useful in the sensor-to-sensor framework.
  - The IV method could handle EIV (closed loop) and inverse model estimation without lossing any performance.
- Future work
  - Working with advanced version of IV method in the inverse model framework
  - Using inverse model framework for fault detection.

Thanks for your attention!

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