Optimization guidance strategies for constrained UAVs
Applications for target tracking of ground-marine vehicle

Alessandro Rucco

Control Optimization and Robotics group
Università del Salento, Lecce (Italy)

alessandro.rucco@unisalento.it

Winter School on
Autonomous Unmanned Aerial Systems for Marine and Coastal Monitoring
Porto, Portugal, January 19, 2018
Few words about me

Alessandro Rucco

- Postdoctoral researcher, Università del Salento (Lecce, Italy) since Oct 2016
- Investigador Auxiliar at the University of Porto (Portugal) Aug 2014 - Sept 2016
- PhD degree in Information Engineering from the Università del Salento (Lecce, Italy) in January 2014, PhD Advisor Prof. Giuseppe Notarstefano
- Visiting scholar at the University of Colorado, Boulder, US, hosted by Prof. John Hauser, Jan-Sept 2012
- Team Leader of the VI-RTUS team (Università del Salento) winning the Virtual Formula 2012\(^a\)
- R&D Engineer at M31 SpA, Padua, Italy

\(^a\)International Competition (with more than 90 registered team) organized by the VI-Grade and sponsored by Lamborghini and Vehicle Dynamics International
Motivations

Trajectory optimization tools

- minimum-time trajectories for race cars,
- long distance trajectory optimization for UAVs,
- cooperative motion planning of autonomous vehicles.
Motivations

Trajectory optimization tools

- minimum-time trajectories for race cars,
- long distance trajectory optimization for UAVs,
- cooperative motion planning of autonomous vehicles.
Motivations

Trajectory optimization tools

- minimum-time trajectories for race cars,
- long distance trajectory optimization for UAVs,
- cooperative motion planning of autonomous vehicles.
Motivations

Trajectory optimization tools

- minimum-time trajectories for race cars,
- long distance trajectory optimization for UAVs,
- cooperative motion planning of autonomous vehicles.

Goal:
Develop an optimization framework for motion planning and guidance of autonomous vehicles in dynamic environments.
Motivations

A motion control system architecture

[Diagram of a motion control system architecture with nodes labeled as Trajectory Generator, Desired Trajectory, High-Level Motion Controller, Path Following, Coordination System, Coordination States, Inner-Loop Controller + Vehicle Dynamics, Navigation System, Navigation Data, and Communication Network.]

A. Rucco – Optimization guidance strategies for constrained UAVs – 2018, Porto 3 | 50
Motivations

A motion control system architecture

[A diagram showing the architecture of a motion control system, including components like Trajectory Generator, Path Following, Coordination System, Navigation System, and Communication Network.]

A. Rucco – Optimization guidance strategies for constrained UAVs – 2018, Porto 3 | 50
Optimization and control tool

internal vehicle model

A. Rucco – Optimization guidance strategies for constrained UAVs – 2018, Porto 4 | 50
Optimization and control tool

desired curve → feedback controller → internal vehicle model → current trajectory
Optimization and control tool

- Desired curve
- Feedback controller
- Internal vehicle model
- Feedback control system
- Current trajectory
- NO human behavior
Optimization and control tool

- Desired curve
- Feedback controller
- Internal vehicle model
- Feedback control system
- Cost function (e.g., $L_2$ norm, time)
- Current cost
- Current trajectory
- Optimization step
- $\delta$-reference trajectory to decrease the cost function
- No human behavior

A. Rucco – Optimization guidance strategies for constrained UAVs – 2018, Porto
A. Rucco – Optimization guidance strategies for constrained UAVs – 2018, Porto 4 | 50
Pros:

- feasible trajectories at each iteration
- non-trivial constraints can be easily handled
- numerically robust thanks to feedback
- reduced *computation time* even for very complex dynamics
Pros:

- feasible trajectories at each iteration
- non-trivial constraints can be easily handled
- numerically robust thanks to feedback
- reduced computation time even for very complex dynamics
Pros:

- feasible trajectories at each iteration
- non-trivial constraints can be easily handled
- numerically robust thanks to feedback
- reduced computation time even for very complex dynamics
Pros:

- feasible trajectories at each iteration
- non-trivial constraints can be easily handled
- numerically robust thanks to feedback
- reduced *computation time* even for very complex dynamics
Optimization and control tool

Main features:

- Development of control oriented models
- Dynamics exploration:
  - analysis of equilibria (trim trajectories)
  - optimal control based strategies to enforce a trajectory-tracking or a path-following behaviour
- Analyze vehicle maneuverability and capabilities

Main challenge: due to the dynamics-constrained nonlinear problems, the computation of such maneuvers for autonomous vehicles is specially challenging.
Optimization and control tool

Main features:

- Development of control oriented models
- Dynamics exploration:
  - analysis of equilibria (trim trajectories)
  - optimal control based strategies to enforce a trajectory-tracking or a path-following behaviour
- Analyze vehicle maneuverability and capabilities

Main challenge: due to the dynamics-constrained nonlinear problems, the computation of such maneuvers for autonomous vehicles is specially challenging.
Coordinated Flight Vehicle (CFV)

Main features:

- (single) planar rigid body
- *coordinated turns*
- *state and input constraints*
Coordinated Flight Vehicle (CFV)

Equations of motion:

\[
\begin{align*}
\dot{x} &= v_a \cos \psi \\
\dot{y} &= v_a \sin \psi \\
\dot{\psi} &= g \tan \phi \\
\dot{v}_a &= u_1 \\
\dot{\phi} &= u_2
\end{align*}
\]

\[v_{a\text{min}} \leq v_a \leq v_{a\text{max}}\]
\[|\phi| \leq \phi_{\text{max}}\]
\[|u_1| \leq u_{1\text{max}}\]
\[|u_2| \leq u_{2\text{max}}\]

- \((x, y)\) and \(\psi\) position and heading angle,
- \(v_a\) airspeed,
- \(\phi\) roll angle,
- \(u_1\) and \(u_2\) control inputs.
Planar UAV model

Main features:

- (single) planar rigid body
- coordinated turns
- wind
- simplified drag
- state and input constraints
Equations of motion:

\[
\begin{align*}
\dot{x} &= v_a \cos \psi + w_x \\
\dot{y} &= v_a \sin \psi + w_y \quad v_{a_{\text{min}}} \leq v_a \leq v_{a_{\text{max}}} \\
\dot{\psi} &= \frac{g \tan \phi}{v_g} \\
\dot{v} &= \frac{T - D}{m}
\end{align*}
\]

- \((x, y)\) and \(\psi\) position and heading angle,
- \(v_a\) air-speed, \(v_g\) ground speed,
- \(w_x\) and \(w_y\) wind velocity components,
- \(T\) and \(\phi\) thrust and roll angle (control inputs),
- \(m\) and \(g\) aircraft’s mass and the gravity acceleration
- \(D(v_a, \phi)\) drag force.
3D UAV model

\[
\begin{align*}
\dot{x} &= v_g \cos \chi \cos \gamma, \\
\dot{y} &= v_g \sin \chi \cos \gamma, \\
\dot{z} &= -v_g \sin \gamma, \\
\dot{\nu} &= \frac{u_1 - D}{m} - g \sin \gamma, \\
\dot{\gamma} &= \frac{1}{v_g} \left( \frac{L \cos \phi}{m} - g \cos \gamma \right), \\
\dot{\chi} &= \frac{1}{v_g \cos \gamma} \left( \frac{L \sin \phi \cos(\chi - \psi)}{m} \right), \\
\dot{\phi} &= u_2,
\end{align*}
\]

Main features

- 3D point mass model
- simplified aerodynamics
- control inputs: thrust, roll rate, lift coeff
- state-input constraints

A. Rucco – Optimization guidance strategies for constrained UAVs – 2018, Porto 9 | 50
3D UAV model

\[
\begin{align*}
\dot{x} &= v_g \cos \chi \cos \gamma, \\
\dot{y} &= v_g \sin \chi \cos \gamma, \\
\dot{z} &= -v_g \sin \gamma, \\
\dot{v} &= \frac{u_1 - D}{m} - g \sin \gamma, \\
\dot{\gamma} &= \frac{1}{v_g} \left( \frac{L \cos \phi}{m} - g \cos \gamma \right), \\
\dot{\chi} &= \frac{1}{v_g \cos \gamma} \left( \frac{L \sin \phi \cos(\chi - \psi)}{m} \right), \\
\dot{\phi} &= u_2, \\
L &= \frac{1}{2} \rho v_a^2 S C_L, \\
D &= \frac{1}{2} \rho v_a^2 S C_D, \\
C_D &= C_{D0} + K_{D/L} C_L^2,
\end{align*}
\]

Main features

- 3D point mass model
- simplified aerodynamics
- control inputs: thrust, roll rate, lift coeff
- state-input constraints

A. Rucco – Optimization guidance strategies for constrained UAVs – 2018, Porto 9 | 50
3D UAV model

\[ \dot{x} = v_g \cos \chi \cos \gamma, \]

\[ \dot{y} = v_g \sin \chi \cos \gamma, \]

\[ \dot{z} = -v_g \sin \gamma, \]

\[ \dot{v} = \frac{u_1 - D}{m} - g \sin \gamma, \]

\[ \dot{\gamma} = \frac{1}{v_g} \left( \frac{L \cos \phi}{m} - g \cos \gamma \right), \]

\[ \dot{\chi} = \frac{1}{v_g \cos \gamma} \left( \frac{L \sin \phi \cos (\chi - \psi)}{m} \right), \]

\[ \dot{\phi} = u_2, \]

\[ L = \frac{1}{2} \rho v_a^2 S C_L, \]

\[ D = \frac{1}{2} \rho v_a^2 S C_D, \]

\[ C_D = C_{D0} + K_{D/L} C_L^2, \]

Main features

- 3D point mass model
- simplified aerodynamics
- control inputs: thrust, roll rate, lift coeff
- state-input constraints
3D UAV model

\[ v_{a\text{min}} \leq v \leq v_{a\text{max}}, \]
\[ n_{lf\text{min}} \leq n_{lf} \leq n_{lf\text{max}}, \]
\[ |\phi| \leq \phi_{\text{max}}, \]
\[ |u_1| \leq u_{1\text{max}}, \]
\[ |u_2| \leq u_{2\text{max}}, \]
\[ |u_3| \leq u_{3\text{max}}. \]

Main features

- 3D point mass model
- simplified aerodynamics
- control inputs: thrust, roll rate, lift coeff
- state-input constraints
General Class of Constrained Vehicles

Vehicle modeled as a rigid body subject to external forces-torques:

**Kinematics:**

\[
\begin{align*}
\dot{p} &= R(\varphi)v + d \\
\dot{\varphi} &= J^{-1}_\omega(\varphi)\omega
\end{align*}
\]

- \(\{I\}\) fixed spatial (inertial) frame
- \(\{B\}\) body-fixed frame
- \(p = [x, y, z]^T\) position of \(\{B\}\) wrt \(\{I\}\)
- \(\varphi = [\phi, \theta, \psi]^T\) orientation of \(\{B\}\) wrt \(\{I\}\)
- \(R(\varphi)\) Roll-Pitch-Yaw parametrization

**Dynamics:**

\[
\begin{align*}
\dot{v} &= f_v(\varphi, v, \omega, u, dv) \\
\dot{\omega} &= f_\omega(\omega, u, d\omega)
\end{align*}
\]

- \(\omega = [p, q, r]^T\) angular velocity wrt \(\{B\}\)
- \(J_\omega(\varphi)\) Jacobian matrix mapping \(\dot{\varphi}\) to \(\omega\)
- \(v = [v_x, v_y, v_z]^T\) linear velocity wrt \(\{B\}\)
- \(u\) control inputs
- \(d, dv,\) and \(d\omega\) external disturbances
We take into account state-input constraints in the following form

\[ c_j(\tilde{x}) \leq 0, \quad j = 1, \ldots, n_c, \]

where \( \tilde{x} \) can be a (coupled) state-input variable of the vehicle model, and \( c_j \) is a smooth function that enforces the constraint.

For instance, the smooth function

\[ c(v) = \left( \frac{2v(t) - (v_{max} + v_{min})}{v_{max} - v_{min}} \right)^2 - 1 \]

enforces the constraint

\[ v_{min} \leq v(t) \leq v_{max} \]
General Class of Constrained Vehicles

We take into account state-input constraints in the following form

\[ c_j(\tilde{x}) \leq 0, \quad j = 1, \ldots, n_c, \]

where \( \tilde{x} \) can be a (coupled) state-input variable of the vehicle model, and \( c_j \) is a smooth function that enforces the constraint.

For instance, the smooth function

\[ c(v) = \left( \frac{2v(t) - (v_{\text{max}} + v_{\text{min}})}{v_{\text{max}} - v_{\text{min}}} \right)^2 - 1 \]

enforces the constraint

\[ v_{\text{min}} \leq v(t) \leq v_{\text{max}} \]
Trim trajectories: trajectories of the system that can be performed by use of appropriate constant inputs.

For the 3D UAV model (NO wind), by enforcing $\dot{v}_a = 0$, $v_a = v_{eq}$, $\dot{\psi} = \dot{\psi}_{eq}$, we have

$$
\begin{cases}
\frac{u_1 - D(v_{eq}, \alpha_{eq})}{m} = g \sin \gamma_{eq} \\
L(v_{eq}, \alpha_{eq}) \frac{\cos \phi}{m} = g \cos \gamma_{eq} \\
\dot{\psi}_{eq} = \frac{1}{v_{eq} \cos \gamma_{eq}} \left( \frac{L(v_{eq}, \alpha_{eq}) \sin \phi}{m} \right)
\end{cases}
$$

Three nonlinear equations in three unknowns (i.e., $u_1, \phi, \alpha$).

Prediction-Corrector continuation method to compute the equilibrium manifold.

Analysis of equilibria

**Trim trajectories:** trajectories of the system that can be performed by use of appropriate constant inputs.

For the 3D UAV model (NO wind), by enforcing $\dot{v}_a = 0$, $v_a = v_{eq}$, $\dot{\psi} = \dot{\psi}_{eq}$, we have

$$\begin{align*}
\frac{u_1 - D(v_{eq},\alpha_{eq})}{m} &= g \sin \gamma_{eq} \\
L(v_{eq},\alpha_{eq})\frac{\cos \phi}{m} &= g \cos \gamma_{eq} \\
\dot{\psi}_{eq} &= \frac{1}{v_{eq} \cos \gamma_{eq}} \left( \frac{L(v_{eq},\alpha_{eq}) \sin \phi}{m} \right)
\end{align*}$$

Three nonlinear equations in three unknowns (i.e., $u_1, \phi, \alpha$).

Prediction-Corrector continuation method to compute the equilibrium manifold.

Analysis of equilibria

**Trim trajectories:** trajectories of the system that can be performed by use of appropriate constant inputs.

For the 3D UAV model (NO wind), by enforcing \( \dot{v}_a = 0, \dot{v}_a = v_{eq}, \dot{\psi} = \dot{\psi}_{eq} \), we have

\[
\begin{aligned}
\frac{u_1 - D(v_{eq}, \alpha_{eq})}{m} &= g \sin \gamma_{eq} \\
L(v_{eq}, \alpha_{eq}) \frac{\cos \phi}{m} &= g \cos \gamma_{eq} \\
\dot{\psi}_{eq} &= \frac{1}{v_{eq} \cos \gamma_{eq}} \left( \frac{L(v_{eq}, \alpha_{eq}) \sin \phi}{m} \right)
\end{aligned}
\]

Three nonlinear equations in three unknowns (i.e., \( u_1, \phi, \alpha \)).

Prediction-Corrector continuation method to compute the equilibrium manifold.

3D point-mass model: trimming trajectories

Parameters based on Zagi flying wing vehicle, “Small Unmanned Aircraft Theory and Practice”, Beard and McLain
Optimal control problem: trajectory-tracking

We are interested in nonlinear optimal control problems of the form

$$\min_{x(\cdot), u(\cdot)} \frac{1}{2} \int_0^T \left( \| x(\tau) - x_d(\tau) \|_Q^2 + \| u(\tau) - u_d(\tau) \|_R^2 d\tau \right) + \frac{1}{2} \| x(T) - x_d(T) \|_{P_1}^2$$

subj. to dynamics constraints

state/input constraints

where

$$(x^d(\cdot), u^d(\cdot))$$ is a desired curve,

$T > 0$ is fixed

$Q$, $R$ and $P_1$ are positive definite weighting matrices
Optimal Control Solver: PRONTO

PRojection Operator based Newton method for Trajectory Optimization

(a) trajectory manifold
(b) search direction
(c) line search
(d) update


Model Validation and trajectory exploration

Experimental flight test performed in Bragança (August 2014) by LSTS

X8 delta wing UAV
(http://lsts.fe.up.pt/vehicles/x8)
Model Validation and trajectory exploration

velocity v

A. Rucco – Optimization guidance strategies for constrained UAVs – 2018, Porto 16 | 50
Model Validation and trajectory exploration

roll angle $\phi$

desired curve
optimal traj

A. Rucco – Optimization guidance strategies for constrained UAVs – 2018, Porto 16 | 50
Virtual Target Vehicle (VTV) approach

**Goal:** drive the vehicle to reach and follow a desired path

**Main idea:** define an error vector between UAV and VTV which moves according to a “convenient velocity”.

**Notation:**
- \((x, y)\) and \(\psi\) position and orientation of the actual vehicle,
- \((x_d, y_d)\) and \(\chi_d\) position and heading angle of the VTV,
- \((e_x, e_y)\) error coordinates,
- \(e\chi = \psi - \chi_d\) local heading angle.

Given the arc-length parametrization of the VTV path, we have

\[
\begin{bmatrix}
  x \\
y
\end{bmatrix}
= \begin{bmatrix}
  \bar{x}_d \\
  \bar{y}_d
\end{bmatrix} + R_z(\bar{\chi}_d) \begin{bmatrix}
  e_x \\
e_y
\end{bmatrix}
\]

\[
R_z(\bar{\chi}_d) = \begin{bmatrix}
  \cos \bar{\chi}_d & -\sin \bar{\chi}_d \\
  \sin \bar{\chi}_d & \cos \bar{\chi}_d
\end{bmatrix}
\]

**Note.** The bar symbol indicates that a quantity is expressed as a function of the arc-length rather than time.
Error dynamics for stationary desired paths

We rewrite the UAV model with respect to a new set of coordinates \((x, u) = (e_x, e_y, e_\chi, v_a, s_{vtv}, u_1, u_2, u_3)\):

\[
\begin{align*}
\dot{e}_x &= v_a \cos e_\chi + w_x \cos \bar{\chi}d + w_y \sin \bar{\chi}d - (1 - e_y \bar{\sigma}_d)u_3 \\
\dot{e}_y &= v_a \sin e_\chi - w_x \sin \bar{\chi}d + w_y \cos \bar{\chi}d - e_x \bar{\sigma}_d u_3 \\
\dot{e}_\chi &= \frac{g \tan \phi}{v_g} - \bar{\sigma}_d u_3 \\
\dot{v}_a &= u_1 \\
\dot{s}_{vtv} &= u_3
\end{align*}
\]

Remark

Using the transverse coordinates, \(e_x = 0\). In such a case, \(\dot{s}_{vtv} = \frac{v_a \cos e_\chi}{1 - \bar{\sigma}_d e_y}\), and a singularity appears at \(e_y = \frac{1}{\bar{\sigma}_d}\).

Virtual Target Vehicle approach

- $p = (x, y, z)$ position of the actual vehicle,
- $\bar{p}_{vtv} = (\bar{x}_{vtv}, \bar{y}_{vtv}, \bar{z}_{vtv})$ position of the VTV,
- $e = (e_x, e_y, e_z)$ error coordinates,
- $\bar{R}_{vtv} = [\bar{T}, \bar{N}, \bar{B}]^T$ rotation matrix transforming vectors expressed in the Serret-Frenet frame into the $\{I\}$.

Note. The bar symbol indicates that a quantity is expressed as a function of the arc-length rather than time.
The VTV moves along the desired path according to the new optimization variable $u_{vtv}$ (VTV’s velocity).

Given the arc-length parametrization of the VTV path, we have

$$p(t) = \bar{p}_{vtv} + \bar{R}_{vtv}e(t)$$
Vehicle model with respect to a new set of coordinates

\[ \dot{e} = R_{vtv}^T (Rv + d) - (g_1 + \hat{\omega}_{vtv} e) u_{vtv}, \]
\[ \dot{\varphi} = J^{-1}(\varphi) \omega, \]
\[ \dot{v} = f_v(\varphi, v, \omega, u), \]
\[ \dot{\omega} = f_\omega(\omega, u), \]
\[ \dot{s}_{vtv} = u_{vtv}. \]

Remark

Using *transverse coordinates*, \( e_x = 0 \). In such a case
\[ \dot{s}_{vtv} = \frac{T^T R_v}{1-\sigma_{vtv} e_y} \] and a singularity appears at \( e_y = \frac{1}{\sigma_{vtv}} \).

Optimal control problem: geometric-tracking

Nonlinear least squares trajectory optimization:

$$\min_{x_t(\cdot), u_t(\cdot)} \frac{1}{2} \int_0^T \| x_t(\tau) - x^d_t(\tau) \|_Q^2 + \| u_t(\tau) - u^d_t(\tau) \|_R^2 d\tau + \frac{1}{2} \| x_t(T) - x^d_t(T) \|_{P_1}^2$$

subj. to dynamics constraints

state/input constraints

where

$$(x^d_t(\cdot), u^d_t(\cdot))$$ is a desired curve,

$T > 0$ is fixed,

$Q$, $R$ and $P_1$ are positive definite weighting matrices
Numerical computations: straight line
Numerical computations: loiter
Numerical computations: loiter
Numerical computations: aggressive maneuver
Numerical computations: loiter (in presence of wind)

\[ R(u_{vtv}) = 0.01 \]

\[ R(u_{vtv}) = 10 \]
Loiter: energy-efficient motion planning

\[ x \text{ [m]} \quad y \text{ [m]} \quad \text{wind} \]

\[ x \text{ [m]} \quad y \text{ [m]} \quad \text{wind} \]

\[ x \text{ [m]} \quad y \text{ [m]} \quad \text{wind} \]

⇒ UAV exploits the wind in order to both minimize the error coordinates and the control effort
Optimal control based strategy for the generation of smooth trajectories for fixed-wing UAVs with applications to target tracking of marine vehicles

- The proposed method is based on the Virtual Target Vehicle (VTV) approach
  ⇒ avoid singularities that occur in some moving path following techniques

- The UAV trajectory is based on the motion of an Autonomous Surface Vehicle (ASV) which has an arbitrary motion

- Space-varying wind fields, state and input constraints, and path constraints are taken into account
Planar UAV model

\[
\begin{align*}
\dot{x} &= v_a \cos \psi + w_x \\
\dot{y} &= v_a \sin \psi + w_y \\
\dot{\psi} &= g \tan \phi \\
\dot{v} &= \frac{T - D}{m}
\end{align*}
\]

- \((x, y)\) and \(\psi\) position and heading angle,
- \(v_a\) air-speed, \(v_g\) ground speed,
- \(w_x\) and \(w_y\) wind velocity components,
- \(T\) and \(\phi\) thrust and roll angle (control inputs),
- \(m\) and \(g\) aircraft’s mass and the gravity acceleration
- \(D(v_a, \phi)\) drag force.
Planar UAV model

State-input constraints:

\[ v_{a\text{ min}} \leq v_a \leq v_{a\text{ max}} \]
\[ 0 \leq T \leq T_{\text{max}} \]
\[ |\phi| \leq \phi_{\text{max}} \]

communication constraints

\[
\left( \frac{x - x_a}{r_{\text{max}}} \right)^2 + \left( \frac{y - y_a}{r_{\text{max}}} \right)^2 \leq 1
\]
Target vehicle: planar ASV model

The UAV trajectory is based on the motion of an Autonomous Surface Vehicle (ASV)

\[
\begin{align*}
\dot{x}_s &= v_{dx} \cos \chi_s - v_{dy} \sin \chi_s \\
\dot{y}_s &= v_{dx} \sin \chi_s + v_{dy} \cos \chi_s \\
\dot{\chi}_s &= \omega_d
\end{align*}
\]

• \((x_s, y_s)\) and \(\chi_s\) position and heading angle, 
• \(v_d = [v_{dx}, v_{dy}]^T\) linear velocity expressed in the body frame, 
• \(\omega_d\) angular velocity expressed in the body frame.
The VTV can be described with respect to the ASV motion.

The VTV coordinates are defined with respect to the ASV position:

\[ p_d = p_s + R_z(\chi_s)\vec{p}_d. \]

The coordinates of the UAV can be defined with respect to the position of the VTV:

\[ p = p_d + R_z(\chi_d)e. \]
Error dynamics

Defining $e_\chi = \chi - \chi_d$ and $u_{vtv} = \dot{s}_{vtv}$, the nonlinear system can be written with respect to the new set of coordinates $(x, u) = (e_x, e_y, e_\chi, v_a, s_{vtv}, u_1, u_2, u_{vtv})$ as follows:

\[
\begin{align*}
\dot{e}_x &= v_g \cos e_\chi - \tilde{v}_{dx} \cos \chi_d - \tilde{v}_{dy} \sin \chi_d - u_{vtv} + \tilde{\omega}_d e_y , \\
\dot{e}_y &= v_g \sin e_\chi + \tilde{v}_{dx} \sin \chi_d - \tilde{v}_{dy} \cos \chi_d - \tilde{\omega}_d e_x , \\
\dot{e}_\chi &= \frac{g \tan u_2 v_a (v_a + w^b_x) - u_1 v_g w^b_y}{v_g^3} - \tilde{\omega}_d , \\
\dot{v}_a &= \frac{u_1 - D}{m} , \\
\dot{s}_{vtv} &= u_{vtv} .
\end{align*}
\]
Target tracking of marine vehicles

Motivating scenario:

The ASV is monitoring a given area for collecting ocean data

The UAV is deployed as a wireless communication relay

The UAV needs to loiter with respect to the ASV position
Smooth UAV trajectory generation

More and more applications for UAVs

- surveillance,
- mapping and photography,
- environmental measurements.

**Cons**: limited fuel capacity.
Optimal Rendezvous Trajectory for UAV-UGV

More and more applications for UAVs

- surveillance,
- mapping and photography,
- environmental measurements.

**Cons**: limited fuel capacity.

Unmmaned Ground Vehicle (UGV) can be deployed for refueling
UAV model: 3D point mass model

\[
\begin{align*}
\dot{x}_A &= v_A \cos \chi_A \cos \gamma_A, \\
\dot{y}_A &= v_A \sin \chi_A \cos \gamma_A, \\
\dot{z}_A &= -v_A \sin \gamma_A, \\
\dot{v}_A &= \frac{u_1 - D}{m} - g \sin \gamma_A, \\
\dot{\gamma}_A &= \frac{1}{v_A} \left( \frac{L \cos \phi_A}{m} - g \cos \gamma_A \right), \\
\dot{\chi}_A &= \frac{1}{v_A \cos \gamma_A} \left( \frac{L \sin \phi_A \cos (\chi_A - \psi_A)}{m} \right), \\
\dot{\phi}_A &= u_2,
\end{align*}
\]
UAV model: 3D point mass model

\[ \dot{x}_A = v_A \cos \chi_A \cos \gamma_A, \]
\[ \dot{y}_A = v_A \sin \chi_A \cos \gamma_A, \]
\[ \dot{z}_A = -v_A \sin \gamma_A, \]
\[ \dot{v}_A = \frac{u_1 - D}{m} - g \sin \gamma_A, \]
\[ \dot{\gamma}_A = \frac{1}{v_A} \left( \frac{L \cos \phi_A}{m} - g \cos \gamma_A \right), \]
\[ \dot{\chi}_A = \frac{1}{v_A \cos \gamma_A} \left( \frac{L \sin \phi_A \cos(\chi_A - \psi_A)}{m} \right) \]
\[ \dot{\phi}_A = u_2, \]
\[ L = \frac{1}{2} \rho v_A^2 S C_L, \quad D = \frac{1}{2} \rho v_A^2 S C_D, \quad C_D = C_{D0} + K_{D/L} C_L^2, \]
UAV model: 3D point mass model

\[ \begin{align*}
\dot{x}_A &= v_A \cos \chi_A \cos \gamma_A, \\
\dot{y}_A &= v_A \sin \chi_A \cos \gamma_A, \\
\dot{z}_A &= -v_A \sin \gamma_A, \\
\dot{v}_A &= \frac{u_1 - D}{m} - g \sin \gamma_A, \\
\dot{\gamma}_A &= \frac{1}{v_A} \left( \frac{L \cos \phi_A}{m} - g \cos \gamma_A \right), \\
\dot{\chi}_A &= \frac{1}{v_A \cos \gamma_A} \left( \frac{L \sin \phi_A \cos(\chi_A - \psi_A)}{m} \right), \\
\dot{\phi}_A &= u_2, \\
L &= \frac{1}{2} \rho v_A^2 S C_L, \\
D &= \frac{1}{2} \rho v_A^2 S C_D, \\
C_D &= C_{D0} + K_{D/L} C_L^2,
\end{align*} \]
UGV model: 2D point mass model

\[ \dot{x}_G = v_G \cos \psi_G , \]
\[ \dot{y}_G = v_G \sin \psi_G , \]
\[ \dot{v}_G = a_{lon} , \]
\[ v_G \dot{\psi}_G = a_{lat} , \]

Main features

- 2D point mass model
- fixed path
- control input: longitudinal acceleration
- state-input constraints
UGV model: 2D point mass model

\[
\begin{align*}
\dot{x}_G &= v_G \cos \psi_G, \\
\dot{y}_G &= v_G \sin \psi_G, \\
\dot{v}_G &= a_{lon}, \\
v_G \dot{\psi}_G &= a_{lat},
\end{align*}
\]

Main features

- 2D point mass model
- fixed path
- control input: longitudinal acceleration
- state-input constraints
UGV model: 2D point mass model

\[ \begin{align*}
\dot{x}_G &= v_G \cos \psi_G , \\
\dot{y}_G &= v_G \sin \psi_G , \\
\dot{v}_G &= u_4 , \\
v_G \dot{\psi}_G &= a_{lat} ,
\end{align*} \]

Main features
- 2D point mass model
- fixed path
- control input: longitudinal acceleration
- state-input constraints
UGV model: 2D point mass model

\[(u_4 + a_{lat})^2 \leq a_{max}^2\]

Main features

- 2D point mass model
- fixed path
- \textit{control input}: longitudinal acceleration
- \textit{state-input constraints}
The coordinates of the Aerial vehicle can be defined as

\[ p_A = p_G + R_z(\psi_G)e \]

**Notation:**

- \( p_A \) position of the Aerial vehicle
- \( p_G \) position of the Ground vehicle
- \( e = (e_x, e_y, e_z) \) error coordinates
- \( R_z(\psi_G) \) rotation matrix
Error dynamics

The coordinates of the Aerial vehicle can be defined as

\[ p_A = p_G + R_z(\psi_G)e \]

Notation:
- \( p_A \) position of the Aerial vehicle
- \( p_G \) position of the Ground vehicle
- \( e = (e_x, e_y, e_z) \) error coordinates
- \( R_z(\psi_G) \) rotation matrix

Error space frames.

\[ \dot{e} = R_z(\chi_G)^T R_z(\chi_A) R_y(\gamma_A) \begin{bmatrix} v_A \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} (1 - e_y \sigma_G)v_G \\ e_x \sigma_G v_G \\ 0 \end{bmatrix}. \]
Constraints for successful rendezvous

Given the coupled UAV-UGV dynamics

\[ e = R_z(\psi_G)^\top (p_A - p_G) \]

\[ e_\chi = \chi_A - \chi_G \]

- \( e_z \leq 0 \) (avoid collision)
- \( |e_\chi| \leq \left( \frac{e_x}{\bar{e}_x} \right)^2 + \left( \frac{e_y}{\bar{e}_y} \right)^2 + \left( \frac{e_z}{\bar{e}_z} \right)^2 + \bar{e}_\chi \) (physical docking)
We now address the problem of computing rendezvous trajectories by using nonlinear least squares trajectory optimization:

\[
\min_{x(\cdot), u(\cdot)} \frac{1}{2} \int_0^T \|x(t) - x^d(t)\|^2_Q + \|u(t) - u^d(t)\|^2_R dt + \frac{1}{2} \|x(T) - x^d(T)\|^2_{P_1}
\]

subj. to dynamics constraints

state/input constraints

where \((x^d(\cdot), u^d(\cdot))\) is a desired curve, \(T > 0\) is fixed, and \(Q, R\) and \(P_1\) are positive definite weighting matrices.
Optimal control problem: trajectory-tracking

\[
\min_{x(\cdot), u(\cdot)} \frac{1}{2} \int_0^T \|x(t) - x^d(t)\|^2_Q + \|u(t) - u^d(t)\|^2_R dt + \frac{1}{2} \|x(T) - x^d(T)\|^2_{P_1}
\]

subj. to dynamics constraints

state/input constraints

**Goal:** compute the optimal trajectory that is close in the \( L_2 \) norm to the desired curve \((x^d(\cdot), u^d(\cdot))\) (path and velocity).

Desired trajectory:

- \( e^d_x, e^d_y, e^d_z, \) and \( \mu^d \) are set to zero.
- \( v^d_A = v^d_G = v_A \min \).
- \( \gamma^d_A \) and \( \phi^d_A \) are set to zero.
- \( u_1^d = T_{eq}(v^d_A), u_2^d, u_3^d, u_4^d \) are set to zero.
Optimal control problem: trajectory-tracking

\[
\min_{x(\cdot), u(\cdot)} \frac{1}{2} \int_0^T \|x(t) - x^d(t)\|_Q^2 + \|u(t) - u^d(t)\|_R^2 dt + \frac{1}{2} \|x(T) - x^d(T)\|_{P_1}^2
\]

subj. to dynamics constraints

state/input constraints

**Remark**

\(Q\) and \(R\) allow us to tune specific performance features of the UAV. By a suitable choice of the weighting matrices, the trajectory can be highly aggressive or it can be very smooth.

Aggressive trajectory
Soft trajectory

![Graphs showing desired and optimal trajectories in 3D space over time, with axes labeled x [m], y [m], z [m], and time [s].]
Rendezvous strategy: main features

- aggressiveness index based on the maximum UAV capability
- desired state-input curve based on the decoupled UAV-UGV
Rendezvous strategy on a turn

Conclusions

Optimization framework for motion planning of UAVs in dynamic environments

Optimal control based strategies to enforce trajectory-tracking or path-following behaviour

Novel approach for generating smooth UAV trajectories (application to target tracking of marine vehicles)

Strategy which allows to select the type of UAV trajectory (application to UAV-UGV rendezvous)
Conclusions

Thank you!