Optimization guidance strategies for constrained UAVs
Applications for target tracking of ground-marine vehicle

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Winter School on
Autonomous Unmanned Aerial Systems for Marine and Coastal Monitoring

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Few words about me

Alessandro Rucco

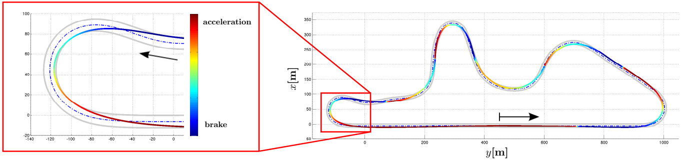
- Postdoctoral researcher, Università del Salento (Lecce, Italy) since Oct 2016
- Investigador Auxiliar at the University of Porto (Portugal) Aug 2014 - Sept 2016
- PhD degree in Information Engineering from the Università del Salento (Lecce, Italy) in January 2014, PhD Advisor Prof. Giuseppe Notarstefano
- Visiting scholar at the University of Colorado, Boulder, US, hosted by Prof. John Hauser, Jan-Sept 2012
- Team Leader of the VI-RTUS team (Università del Salento) winning the Virtual Formula 2012[a]
- R&D Engineer at M31 SpA, Padua, Italy

[a] International Competition (with more than 90 registered team) organized by the VI-Grade and sponsored by Lamborghini and Vehicle Dynamics International

Motivations

Trajectory optimization tools

- minimum-time trajectories for race cars,
- long distance trajectory optimization for UAVs,
- cooperative motion planning of autonomous vehicles.



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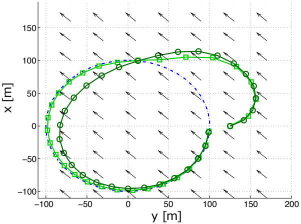

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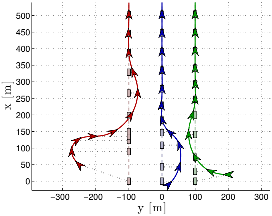

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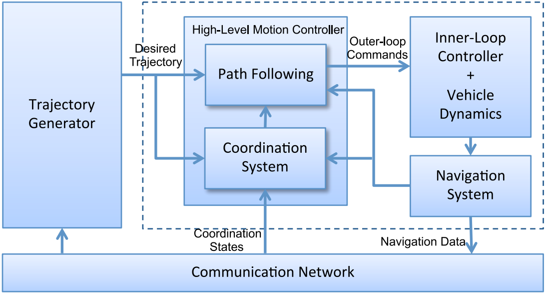
Goal:
Develop an optimization framework for motion planning and guidance of autonomous vehicles in dynamic environments.

Motivations

A motion control system architecture



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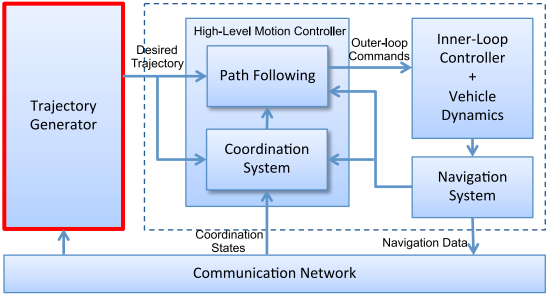
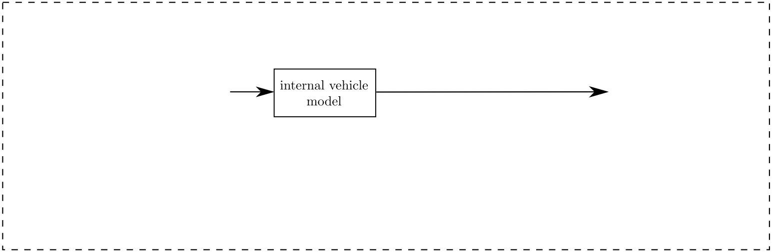


Optimization and control tool



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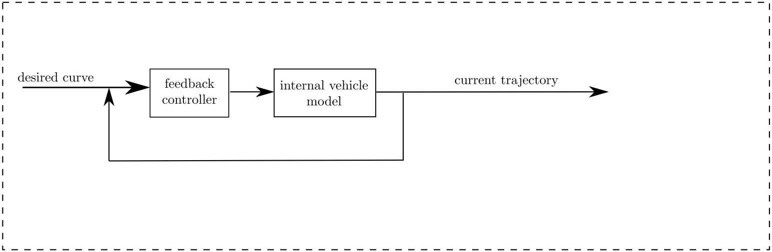

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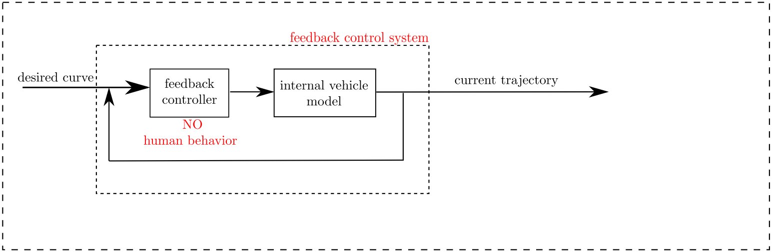

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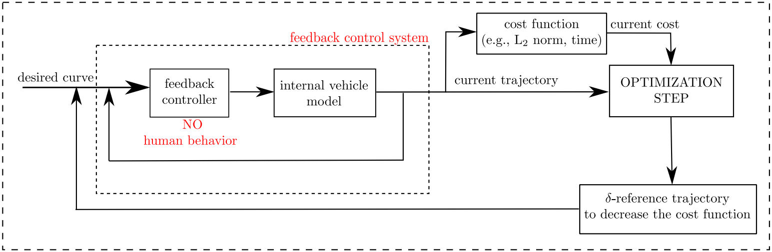

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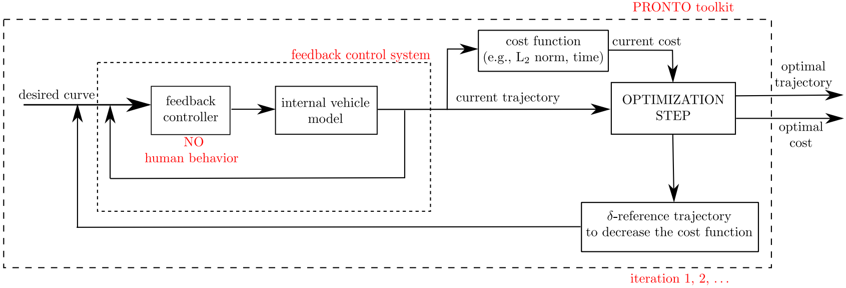

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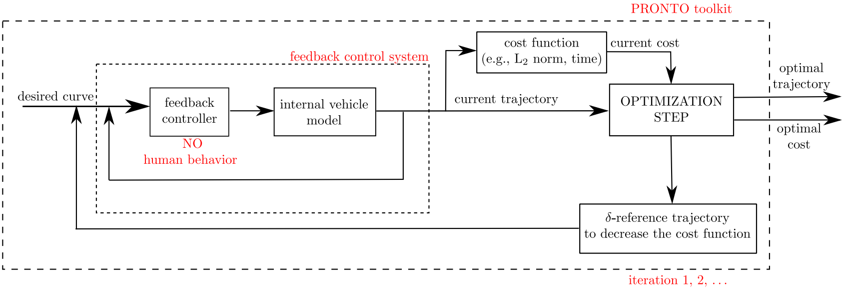
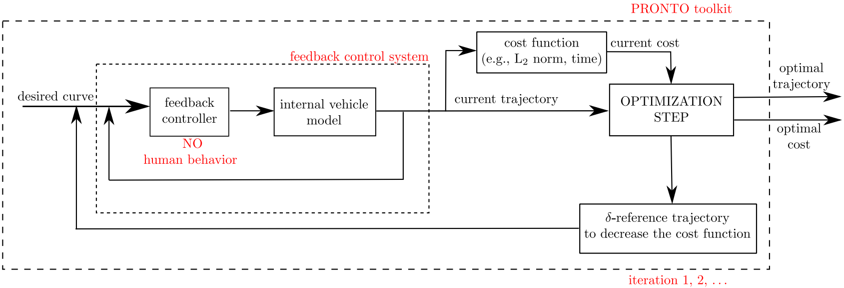


Pros:

- feasible trajectories at each iteration
- non-trivial constraints can be easily handled
- numerically robust thanks to feedback
- reduced *computation time* even for very complex dynamics

Optimization and control tool



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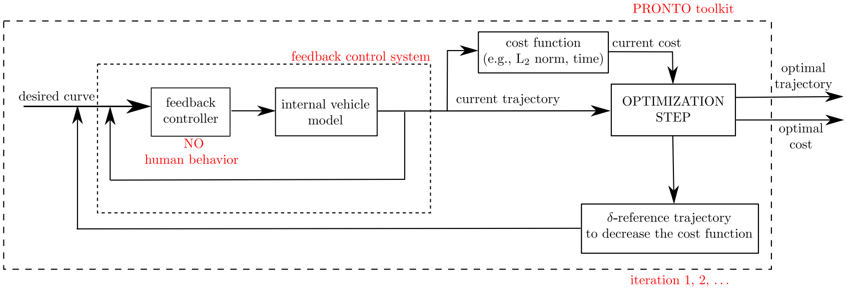

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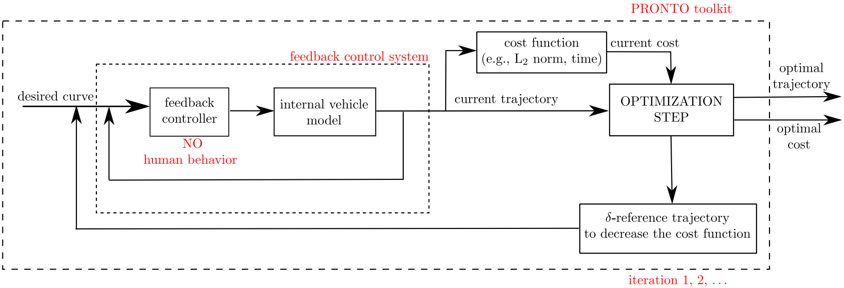

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Optimization and control tool

Main features:

- Development of control oriented models
- Dynamics exploration:
  - analysis of equilibria (trim trajectories)
  - optimal control based strategies to enforce a *trajectory-tracking* or a *path-following* behaviour
- Analyze *vehicle maneuverability and capabilities*

Main challenge: due to the dynamics-constrained nonlinear problems, the computation of such maneuvers for autonomous vehicles is specially challenging.

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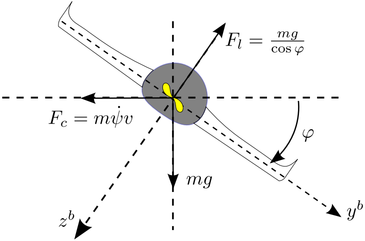
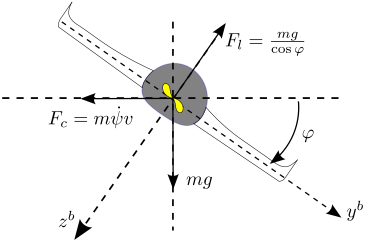
Coordinated Flight Vehicle (CFV)



Main features:

- (single) planar rigid body
- *coordinated turns*
- *state and input constraints*

Coordinated Flight Vehicle (CFV)



Equations of motion:

$$\dot{x} = v_a \cos\psi$$

$$\dot{y} = v_a \sin\psi \qquad v_{a_{min}} \leq v_a \leq v_{a_{max}}$$

$$\dot{\psi} = \frac{g \tan\phi}{v_a} \qquad |\phi| \leq \phi_{max}$$

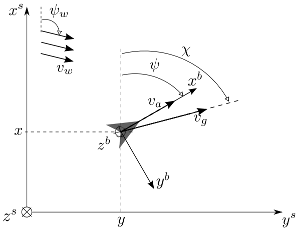
$$|u_1| \leq u_{1max}$$

$$\dot{v}_a = u_1 \qquad |u_2| \leq u_{2max}$$

$$\dot{\phi} = u_2$$

- (x, y) and ψ position and heading angle,
- v_a airspeed,
- ϕ roll angle,
- u_1 and u_2 control inputs.

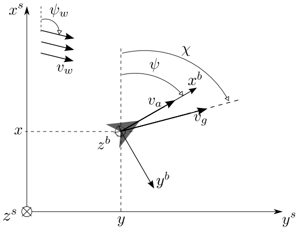
Planar UAV model



Main features:

- (single) planar rigid body
- coordinated turns
- *wind*
- *simplified drag*
- *state and input constraints*

Planar UAV model



Equations of motion:

$$\dot{x} = v_a \cos\psi + w_x$$

$$\dot{y} = v_a \sin\psi + w_y \quad v_{a_{min}} \leq v_a \leq v_{a_{max}}$$

$$\dot{\psi} = \frac{g \tan\phi}{v_g} \quad |T| \leq T_{max}$$

$$|\phi| \leq \phi_{max}$$

$$\dot{v} = \frac{T - D}{m}$$

- (x, y) and ψ position and heading angle,
- v_a air-speed, v_g ground speed,
- w_x and w_y wind velocity components,
- T and ϕ thrust and roll angle (control inputs),
- m and g aircraft's mass and the gravity acceleration
- $D(v_a, \phi)$ drag force.

3D UAV model

$$
\begin{aligned}
\dot{x} &= v_g \cos\chi \cos\gamma, \\
\dot{y} &= v_g \sin\chi \cos\gamma, \\
\dot{z} &= -v_g \sin\gamma, \\
\dot{v} &= \frac{u_1 - D}{m} - g\sin\gamma, \\
\dot{\gamma} &= \frac{1}{v_g}\left(\frac{L\cos\phi}{m} - g\cos\gamma\right), \\
\dot{\chi} &= \frac{1}{v_g\cos\gamma}\left(\frac{L\sin\phi\cos(\chi-\psi)}{m}\right), \\
\dot{\phi} &= u_2,
\end{aligned}
$$

Main features

- 3D point mass model
- *simplified aerodynamics*
- *control inputs*: thurst, roll rate, lift coeff
- *state-input constraints*

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L &= \frac{1}{2}\rho v_a^2 S C_L, D = \frac{1}{2}\rho v_a^2 S C_D, C_D = C_{D_0} + K_{D/L} C_L^2,
\end{aligned}
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3D UAV model

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\begin{aligned}
v_{a_{min}} \leq v \leq v_{a_{max}} \,, \\
n_{lf\,min} \leq n_{lf} \leq n_{lf\,max} \,, \\
|\phi| \leq \phi_{max} \,, \\
|u_1| \leq u_{1max} \,, \\
|u_2| \leq u_{2max} \,, \\
|u_3| \leq u_{3max} \,.
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General Class of Constrained Vehicles

Vehicle modeled as a rigid body subject to external forces-torques:

Kinematics:

$$\boxed{\begin{aligned}\dot{p} &= R(\varphi)v + d \\ \dot{\varphi} &= J_\omega^{-1}(\varphi)\omega\end{aligned}}$$

Dynamics:

$$\boxed{\begin{aligned}\dot{v} &= f_v(\varphi, v, \omega, u, d_v) \\ \dot{\omega} &= f_\omega(\omega, u, d_\omega)\end{aligned}}$$

- $\{\mathcal{I}\}$ fixed spatial (inertial) frame
- $\{\mathcal{B}\}$ body-fixed frame
- $p = [x, y, z]^T$ position of $\{\mathcal{B}\}$ wrt $\{\mathcal{I}\}$
- $\varphi = [\phi, \theta, \psi]^T$ orientation of $\{\mathcal{B}\}$ wrt $\{\mathcal{I}\}$
- $R(\varphi)$ Roll-Pitch-Yaw parametrization
- $\omega = [p, q, r]^T$ angular velocity wrt $\{\mathcal{B}\}$
- $J_\omega(\varphi)$ Jacobian matrix mapping $\dot{\varphi}$ to ω
- $v = [v_x, v_y, v_z]^T$ linear velocity wrt $\{\mathcal{B}\}$
- u control inputs
- d, d_v, and d_ω external disturbances

General Class of Constrained Vehicles

We take into account state-input constraints in the following form

$$c_j(\tilde{x}) \leq 0\,, \quad j = 1, \ldots, n_c\,,$$

where $\tilde{x}$ can be a (coupled) state-input variable of the vehicle model, and c_j is a smooth function that enforces the constraint.

For instance, the smooth function

$$c(v) = \left(\frac{2v(t) - (v_{max} + v_{min})}{v_{max} - v_{min}}\right)^2 - 1$$

enforces the constraint

$$v_{min} \leq v(t) \leq v_{max}$$

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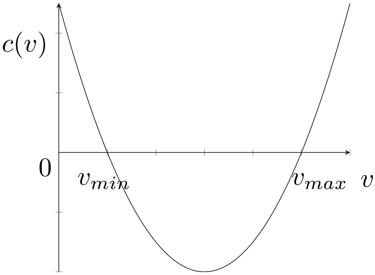

Analysis of equilibria

Trim trajectories: trajectories of the system that can be performed by use of appropriate constant inputs.

For the 3D UAV model (NO wind), by enforcing $\dot{v}_a = 0$, $v_a = v_{eq}$, $\dot{\psi} = \dot{\psi}_{eq}$, we have

$$\begin{cases} \frac{u_1 - D(v_{eq}, \alpha_{eq})}{m} = g \sin \gamma_{eq} \\ L(v_{eq}, \alpha_{eq}) \frac{\cos \phi}{m} = g \cos \gamma_{eq} \\ \dot{\psi}_{eq} = \frac{1}{v_{eq} \cos \gamma_{eq}} \left(\frac{L(v_{eq}, \alpha_{eq}) \sin \phi}{m} \right) \end{cases}$$

Three nonlinear equations in three unknowns (i.e., u_1, ϕ, α).

Prediction-Corrector continuation method to compute the equilibrium manifold.

A. Rucco, G. Notarstefano, J. Hauser, "Optimal Control Based Dynamics Exploration of a Rigid Car With Longitudinal Load Transfer," IEEE TCST 2014.

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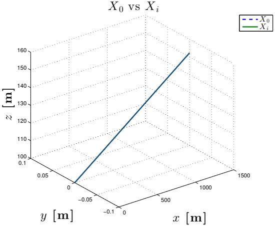
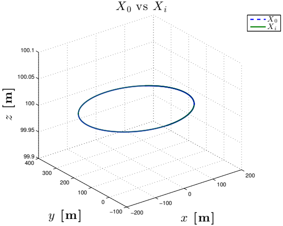
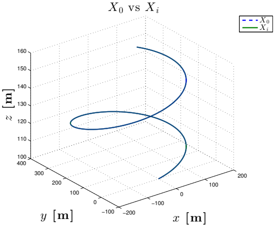
3D point-mass model: trimming trajectories







Parameters based on Zagi flying wing vehicle, "Small Unmanned Aircraft Theory and Practice", Beard and McLain

Optimal control problem: trajectory-tracking

We are interested in nonlinear optimal control problems of the form

$$\min_{x(\cdot),u(\cdot)} \frac{1}{2}\int_0^T \left(\|x(\tau)-x_d(\tau)\|_Q^2 + \|u(\tau)-u_d(\tau)\|_R^2 d\tau\right) + \frac{1}{2}\|x(T)-x_d(T)\|_{P_1}^2$$

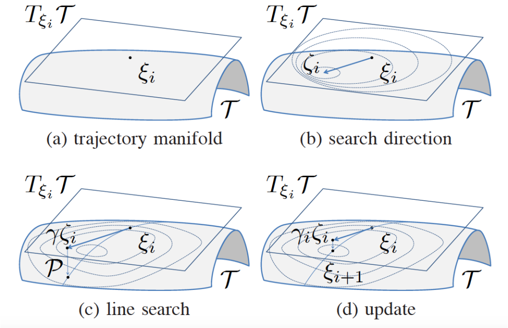
subj. to *dynamics constraints*

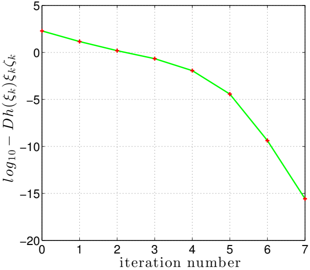
state/input constraints

where
$(x^d(\cdot), u^d(\cdot))$ is a desired curve,
$T > 0$ is fixed
Q, R and P_1 are positive definite weighting matrices

Optimal Control Solver: PRONTO

PRojection Operator based Newton method for Trajectory Optimization

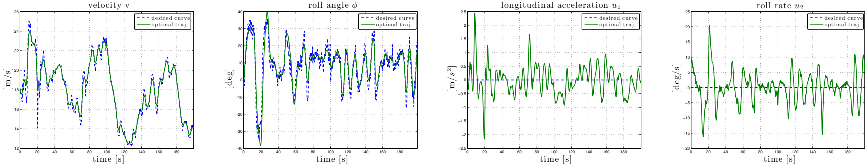


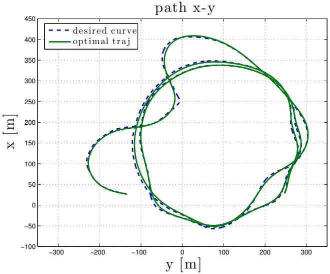


J. Hauser, "A projection operator approach to the optimization of trajectory functionals," in IFAC World Congress, Barcelona, 2002.

A. Saccon, J. Hauser, A. P. Aguiar, "Optimal Control on Lie Groups: The Projection Operator Approach," IEEE Transactions on Automatic Control, 2013

Model Validation and trajectory exploration

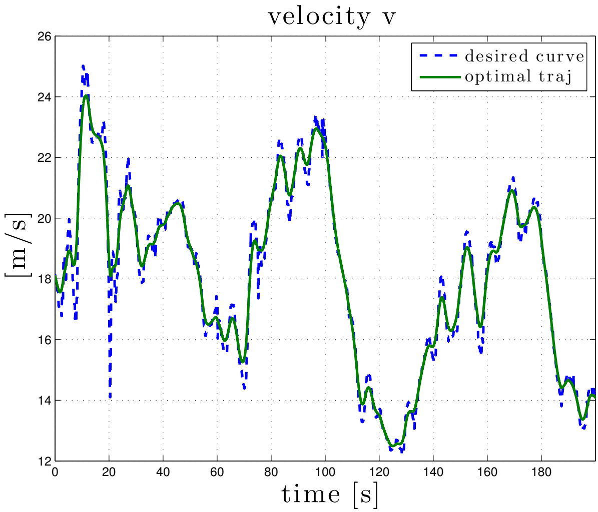


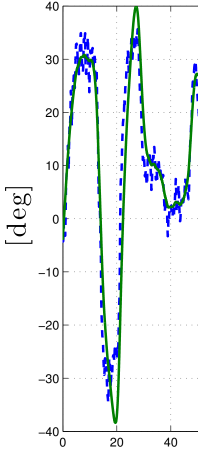


X8 delta wing UAV
(*http://lsts.fe.up.pt/vehicles/x8*)

Experimental flight test performed in Bragança (August 2014) by LSTS

Model Validation and trajectory exploration





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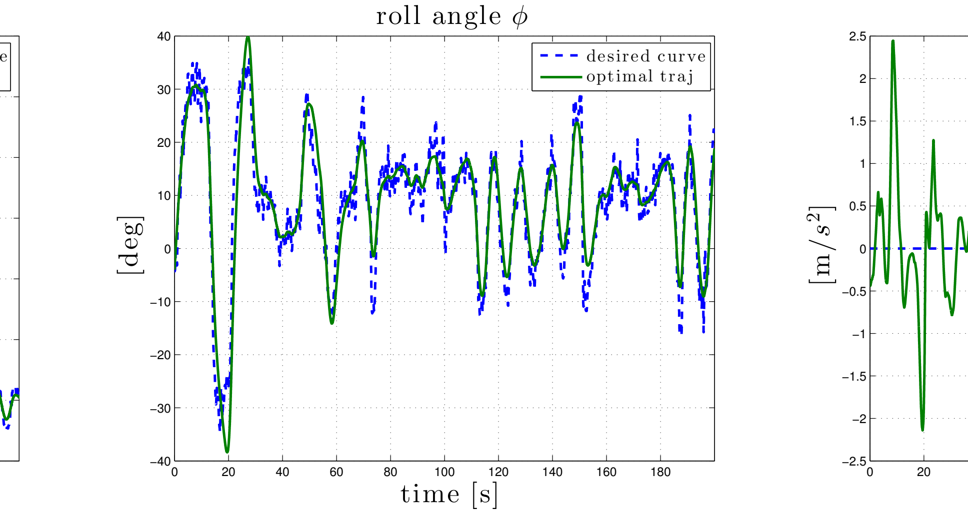
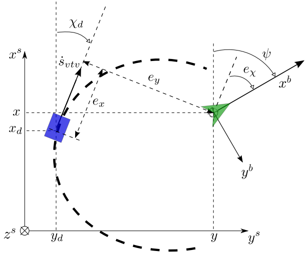


Virtual Target Vehicle (VTV) approach

Goal: drive the vehicle to reach and follow a desired path

Main idea: define an error vector between UAV and VTV which moves according to a “convenient velocity”.

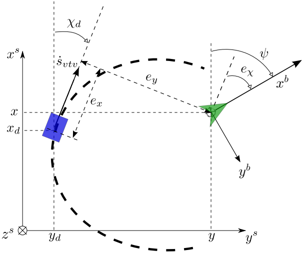


Notation:

- (x, y) and ψ position and orientation of the actual vehicle,
- (x_d, y_d) and χ_d position and heading angle of the VTV,
- (e_x, e_y) error coordinates,
- $e_\chi = \psi - \chi_d$ local heading angle.

L. Lapierre, D. Soetanto, A. Pascoal, “Nonlinear Path Following with Applications to the Control of Autonomous Underwater Vehicles”, in IEEE Conf. on Decision and Control, 2003.

Error dynamics for stationary desired paths



Given the arc-length parametrization of the VTV path, we have

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \bar{x}_d \\ \bar{y}_d \end{bmatrix} + R_z(\bar{\chi}_d) \begin{bmatrix} e_x \\ e_y \end{bmatrix}$$

$$R_z(\bar{\chi}_d) = \begin{bmatrix} \cos\bar{\chi}_d & -\sin\bar{\chi}_d \\ \sin\bar{\chi}_d & \cos\bar{\chi}_d \end{bmatrix}$$

Note. The bar symbol indicates that a quantity is expressed as a function of the arc-length rather than time.

Error dynamics for stationary desired paths

We rewrite the UAV model with respect to a new set of coordinates $(\mathbf{x}, \mathbf{u}) = (e_x, e_y, e_\chi, v_a, s_{vtv}, u_1, u_2, u_3)$:

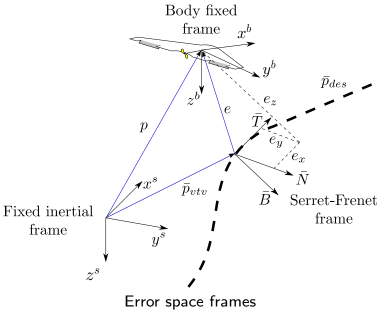
$$
\begin{aligned}
\dot{e}_x &= v_a \cos e_\chi + w_x \cos \bar{\chi}_d + w_y \sin \bar{\chi}_d - (1 - e_y \bar{\sigma}_d) u_3 \\
\dot{e}_y &= v_a \sin e_\chi - w_x \sin \bar{\chi}_d + w_y \cos \bar{\chi}_d - e_x \bar{\sigma}_d u_3 \\
\dot{e}_\chi &= \frac{g \tan \phi}{v_g} - \bar{\sigma}_d u_3 \\
\dot{v}_a &= u_1 \\
\dot{s}_{vtv} &= u_3
\end{aligned}
$$

Remark

Using the *transverse coordinates*, $e_x = 0$. In such a case, $\dot{s}_{vtv} = \frac{v_a \cos e_\chi}{1 - \bar{\sigma}_d e_y}$, and a singularity appears at $e_y = \frac{1}{\bar{\sigma}_d}$.

A. Rucco, A.P. Aguiar, J.Hauser, "Trajectory Optimization for Constrained UAVs: a Virtual Target Vehicle Approach", in 2015 International Conference on Unmanned Aircraft Systems (ICUAS), June 2015.

Virtual Target Vehicle approach

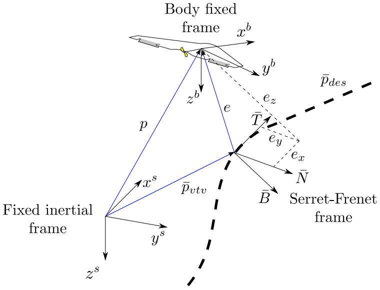


- $p = (x, y, z)$ position of the actual vehicle,
- $\bar{p}_{vtv} = (\bar{x}_{vtv}, \bar{y}_{vtv}, \bar{z}_{vtv})$ position of the VTV,
- $e = (e_x, e_y, e_z)$ error coordinates,
- $\bar{R}_{vtv} = [\bar{T}, \bar{N}, \bar{B}]^T$ rotation matrix transforming vectors expressed in the Serret-Frenet frame into the $\{\mathcal{I}\}$.

Note. The bar symbol indicates that a quantity is expressed as a function of the arc-length rather than time.

Virtual Target Vehicle approach

The VTV moves along the desired path according to the new optimization variable u_{vtv} (VTV's velocity).



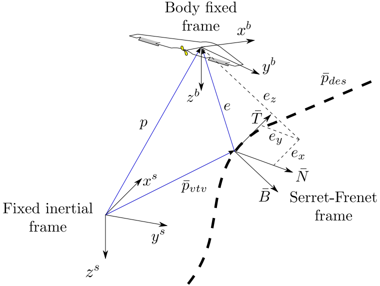
Error space frames

Given the arc-length parametrization of the VTV path, we have

$$p(t) = \bar{p}_{vtv} + \bar{R}_{vtv} e(t)$$

Virtual Target Vehicle approach

Vehicle model with respect to a new set of coordinates



$$
\begin{aligned}
\dot{e} &= \bar{R}_{vtv}^{T}(Rv + d) - (g_1 + \hat{\omega}_{vtv} e) u_{vtv}\,, \\
\dot{\varphi} &= J_{\omega}^{-1}(\varphi)\omega\,, \\
\dot{v} &= f_v(\varphi, v, \omega, u)\,, \\
\dot{\omega} &= f_{\omega}(\omega, u)\,, \\
\dot{s}_{vtv} &= u_{vtv}\,.
\end{aligned}
$$

Remark

Using *transverse coordinates*, $e_x = 0$. In such a case $\dot{s}_{vtv} = \frac{\bar{T}^T Rv}{1-\bar{\sigma}_{vtv} e_y}$ and a singularity appears at $e_y = \frac{1}{\bar{\sigma}_{vtv}}$.

A. Rucco, A.P. Aguiar, J.Hauser, "A virtual target approach for trajectory optimization of a general class of constrained vehicles", CDC, 2015.

Optimal control problem: geometric-tracking

Nonlinear least squares trajectory optimization:

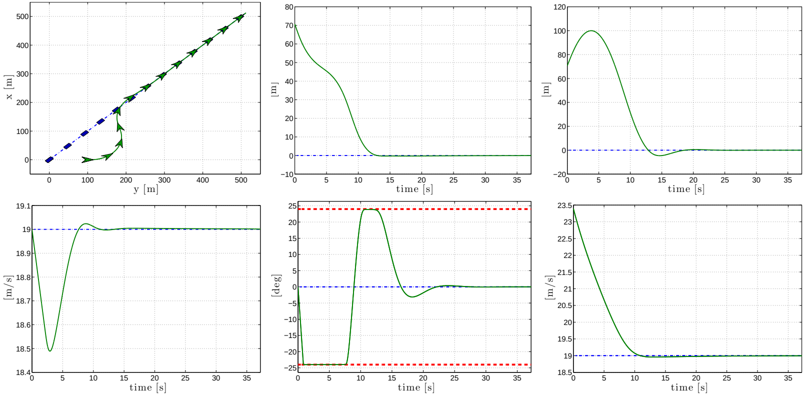
$$\min_{\mathbf{x}_t(\cdot),\mathbf{u}_t(\cdot)} \frac{1}{2}\int_0^T \|\mathbf{x}_t(\tau)-\mathbf{x}_t^d(\tau)\|_Q^2+\|\mathbf{u}_t(\tau)-\mathbf{u}_t^d(\tau)\|_R^2 d\tau + \frac{1}{2}\|\mathbf{x}_t(T)-\mathbf{x}_t^d(T)\|_{P_1}^2$$

subj. to *dynamics constraints*

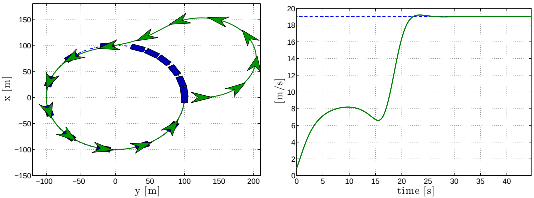
state/input constraints

where
$(\mathbf{x}_t^d(\cdot), \mathbf{u}_t^d(\cdot))$ is a desired curve,
$T > 0$ is fixed,
Q, R and P_1 are positive definite weighting matrices

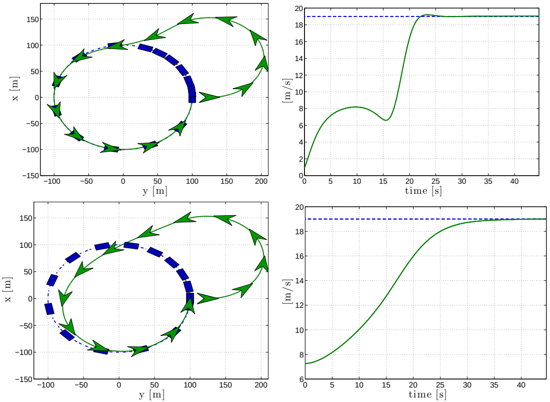
Numerical computations: straight line



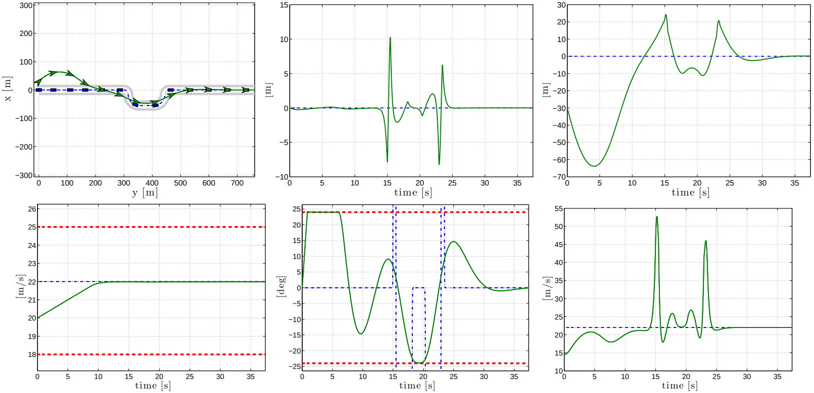
Numerical computations: loiter



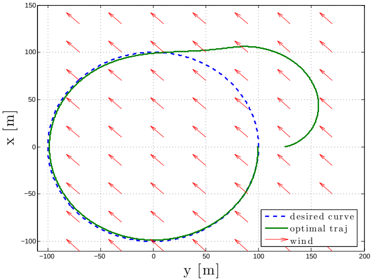
Numerical computations: loiter



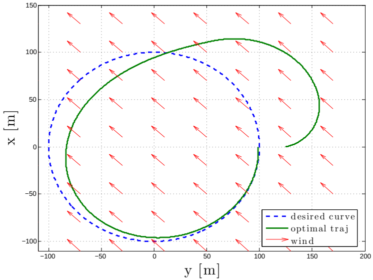
Numerical computations: aggressive maneuver



Numerical computations: loiter (in presence of wind)

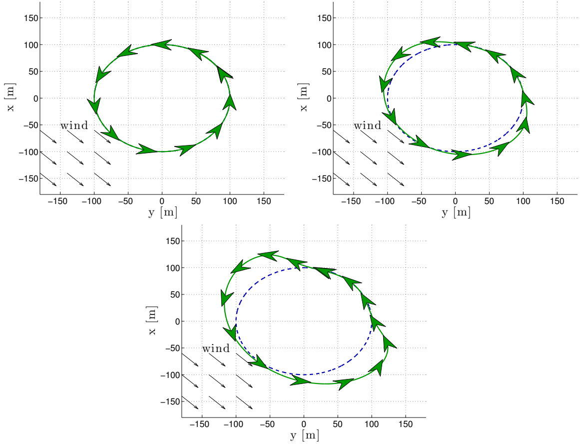


$$R(u_{vtv}) = 0.01$$



$$R(u_{vtv}) = 10$$

Loiter: energy-efficient motion planning



$\Rightarrow$ UAV exploits the wind in order to both minimize the error coordinates and the control effort

Moving Path Following Approach

Optimal control based strategy for the generation of smooth trajectories for fixed-wing UAVs with applications to target tracking of marine vehicles

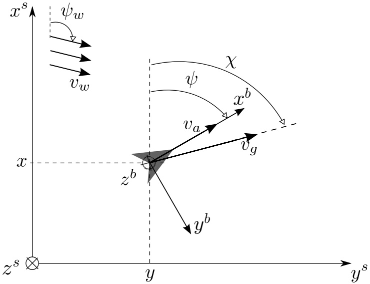
- The proposed method is based on the Virtual Target Vehicle (VTV) approach

  $\Rightarrow$ avoid singularities that occur in some moving path following techniques

- The UAV trajectory is based on the motion of an Autonomous Surface Vehicle (ASV) which has an arbitrary motion

- Space-varying wind fields, state and input constraints, and path constraints are taken into account

Planar UAV model

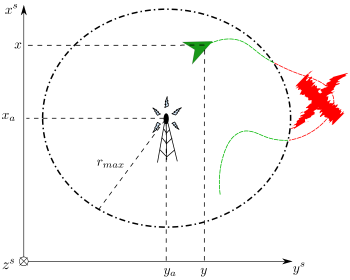


Planar UAV model:

$$\dot{x} = v_a \cos\psi + w_x$$
$$\dot{y} = v_a \sin\psi + w_y$$
$$\dot{\psi} = \frac{g \tan\phi}{v_g}$$
$$\dot{v} = \frac{T - D}{m}$$

- (x, y) and ψ position and heading angle,
- v_a air-speed, v_g ground speed,
- w_x and w_y wind velocity components,
- T and ϕ thrust and roll angle (control inputs),
- m and g aircraft's mass and the gravity acceleration
- $D(v_a, \phi)$ drag force.

Planar UAV model



State-input constraints:

$$v_{a\,min} \leq v_a \leq v_{a\,max}$$
$$0 \leq T \leq T_{max}$$
$$|\phi| \leq \phi_{max}$$

communication constraints

$$\left(\frac{x - x_a}{r_{max}}\right)^2 + \left(\frac{y - y_a}{r_{max}}\right)^2 \leq 1$$

Target vehicle: planar ASV model

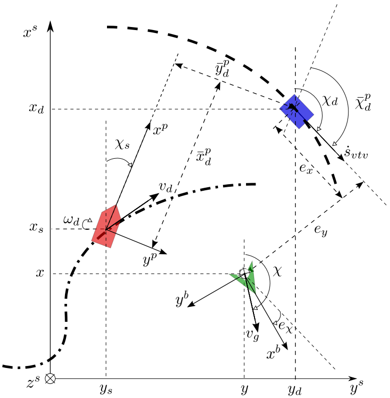
The UAV trajectory is based on the motion of an Autonomous Surface Vehicle (ASV)

$$
\begin{aligned}
\dot{x}_s &= v_{dx}\cos\chi_s - v_{dy}\sin\chi_s\\
\dot{y}_s &= v_{dx}\sin\chi_s + v_{dy}\cos\chi_s\\
\dot{\chi}_s &= \omega_d
\end{aligned}
$$

- (x_s, y_s) and χ_s position and heading angle,
- $v_d = [v_{dx}, v_{dy}]^T$ linear velocity expressed in the body frame,
- ω_d angular velocity expressed in the body frame.

Virtual Target Vehicle approach, moving paths

The VTV can be described with respect to the ASV motion



The VTV coordinates are defined with respect to the ASV position:

$$p_d = p_s + R_z(\chi_s)\bar{p}_d^p .$$

The coordinates of the UAV can be defined with respect to the position of the VTV:

$$p = p_d + R_z(\chi_d)e.$$

Error dynamics

Defining $e_\chi = \chi - \chi_d$ and $u_{vtv} = \dot{s}_{vtv}$, the nonlinear system can be written with respect to the new set of coordinates $(\mathbf{x}, \mathbf{u}) = (e_x, e_y, e_\chi, v_a, s_{vtv}, u_1, u_2, u_{vtv})$ as follows:

$$\dot{e}_x = v_g \cos e_\chi - \tilde{v}_{dx} \cos \chi_d - \tilde{v}_{dy} \sin \chi_d - u_{vtv} + \tilde{\omega}_d e_y \,,$$

$$\dot{e}_y = v_g \sin e_\chi + \tilde{v}_{dx} \sin \chi_d - \tilde{v}_{dy} \cos \chi_d - \tilde{\omega}_d e_x \,,$$

$$\dot{e}_\chi = \frac{g \tan u_2 v_a (v_a + w_x^b) - u_1 v_g w_y^b}{v_g^3} - \tilde{\omega}_d \,,$$

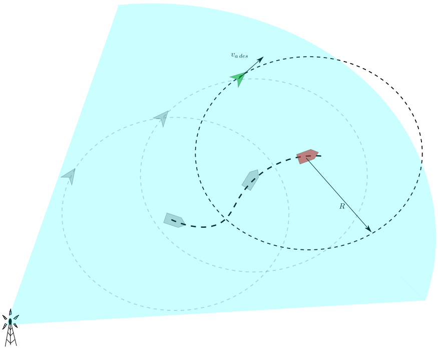
$$\dot{v}_a = \frac{u_1 - D}{m} \,,$$

$$\dot{s}_{vtv} = u_{vtv} \,.$$

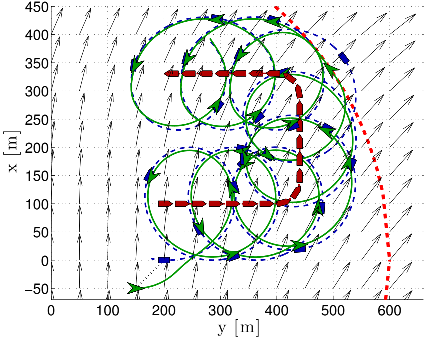
Target tracking of marine vehicles

Motivating scenario:

- The ASV is monitoring a given area for collecting ocean data
- The UAV is deployed as a wireless communication relay
- The UAV needs to loiter with respect to the ASV position



Smooth UAV trajectory generation



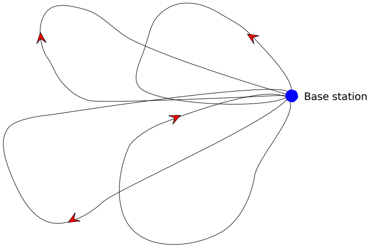
A. Rucco, A.P. Aguiar, F.L. Pereira, and J.B. de Sousa "A Moving Path Following Approach for Trajectory Optimization of UAVs: An application for target tracking of marine vehicles," ECC 2016.

Optimal Rendezvous Trajectory for UAV-UGV

More and more applications for UAVs

- surveillance,
- mapping and photography,
- environmental measurements.

Cons: limited fuel capacity.



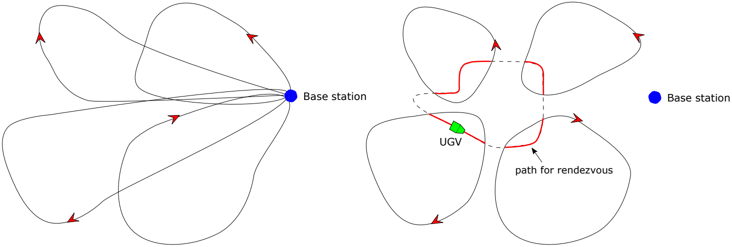
Optimal Rendezvous Trajectory for UAV-UGV

More and more applications for UAVs

- surveillance,
- mapping and photography,
- environmental measurements.

Cons: limited fuel capacity.

Unmmaned Ground Vehicle (UGV) can be deployed for refuling



UAV model: 3D point mass model

$$
\begin{aligned}
\dot{x}_A &= v_A \cos\chi_A \cos\gamma_A, \\
\dot{y}_A &= v_A \sin\chi_A \cos\gamma_A, \\
\dot{z}_A &= -v_A \sin\gamma_A, \\
\dot{v}_A &= \frac{u_1 - D}{m} - g \sin\gamma_A, \\
\dot{\gamma}_A &= \frac{1}{v_A}\left(\frac{L\cos\phi_A}{m} - g\cos\gamma_A\right), \\
\dot{\chi}_A &= \frac{1}{v_A\cos\gamma_A}\left(\frac{L\sin\phi_A\cos(\chi_A - \psi_A)}{m}\right), \\
\dot{\phi}_A &= u_2,
\end{aligned}
$$

UAV model: 3D point mass model

$$
\begin{aligned}
\dot{x}_A &= v_A \cos\chi_A \cos\gamma_A, \\
\dot{y}_A &= v_A \sin\chi_A \cos\gamma_A, \\
\dot{z}_A &= -v_A \sin\gamma_A, \\
\dot{v}_A &= \frac{u_1 - D}{m} - g\sin\gamma_A, \\
\dot{\gamma}_A &= \frac{1}{v_A}\left(\frac{L\cos\phi_A}{m} - g\cos\gamma_A\right), \\
\dot{\chi}_A &= \frac{1}{v_A\cos\gamma_A}\left(\frac{L\sin\phi_A\cos(\chi_A - \psi_A)}{m}\right) \\
\dot{\phi}_A &= u_2, \\
L &= \frac{1}{2}\rho v_A^2 S C_L, D = \frac{1}{2}\rho v_A^2 S C_D, C_D = C_{D_0} + K_{D/L} C_L^2,
\end{aligned}
$$

UAV model: 3D point mass model

$$
\begin{aligned}
\dot{x}_A &= v_A \cos\chi_A \cos\gamma_A, \\
\dot{y}_A &= v_A \sin\chi_A \cos\gamma_A, \\
\dot{z}_A &= -v_A \sin\gamma_A, \\
\dot{v}_A &= \frac{u_1 - D}{m} - g\sin\gamma_A, \\
\dot{\gamma}_A &= \frac{1}{v_A}\left(\frac{L\cos\phi_A}{m} - g\cos\gamma_A\right), \\
\dot{\chi}_A &= \frac{1}{v_A\cos\gamma_A}\left(\frac{L\sin\phi_A\cos(\chi_A - \psi_A)}{m}\right) \\
\dot{\phi}_A &= u_2, \\
L &= \frac{1}{2}\rho v_A^2 S C_L, D = \frac{1}{2}\rho v_A^2 S C_D, C_D = C_{D_0} + K_{D/L} {C_L}^2,
\end{aligned}
$$

UGV model: 2D point mass model

$$
\begin{aligned}
\dot{x}_G &= v_G \cos \psi_G \,, \\
\dot{y}_G &= v_G \sin \psi_G \,, \\
\dot{v}_G &= a_{lon} \,, \\
v_G \dot{\psi}_G &= a_{lat} \,,
\end{aligned}
$$

Main features

- 2D point mass model
- fixed path
- *control input*: longitudinal acceleration
- *state-input constraints*

UGV model: 2D point mass model

$$\dot{x}_G = v_G \cos\psi_G \,,$$
$$\dot{y}_G = v_G \sin\psi_G \,,$$
$$\dot{v}_G = a_{lon} \,,$$
$$v_G \dot{\psi}_G = a_{lat} \,,$$

Main features

- 2D point mass model
- fixed path
- *control input*: longitudinal acceleration
- *state-input constraints*

UGV model: 2D point mass model

$$
\begin{aligned}
\dot{x}_G &= v_G \cos\psi_G \,, \\
\dot{y}_G &= v_G \sin\psi_G \,, \\
\dot{v}_G &= u_4 \,, \\
v_G\dot{\psi}_G &= a_{lat} \,,
\end{aligned}
$$

Main features

- 2D point mass model
- fixed path
- *control input*: longitudinal acceleration
- *state-input constraints*

UGV model: 2D point mass model

$$(u_4 + a_{lat})^2 \leq a_{max}^2$$

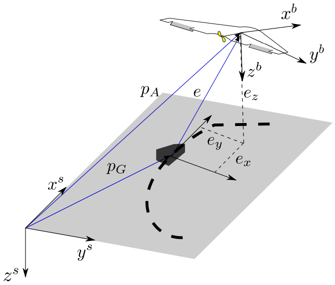
Main features

- 2D point mass model
- fixed path
- *control input*: longitudinal acceleration
- *state-input constraints*

Error dynamics

The coordinates of the Aerial vehicle can be defined as

$$\boxed{p_A = p_G + R_z(\psi_G)e}$$



Error space frames.

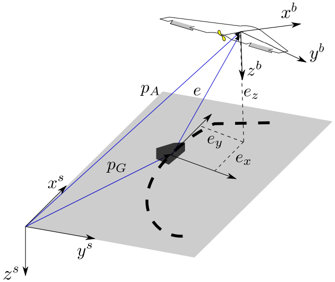
Notation:

- p_A position of the Aerial vehicle
- p_G position of the Ground vehicle
- $e = (e_x, e_y, e_z)$ error coordinates
- $R_z(\psi_G)$ rotation matrix

Error dynamics

The coordinates of the Aerial vehicle can be defined as

$$\boxed{p_A = p_G + R_z(\psi_G)e}$$



Error space frames.

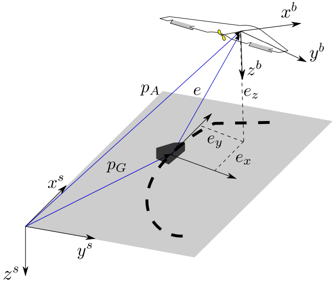
Notation:

- p_A position of the Aerial vehicle
- p_G position of the Ground vehicle
- $e = (e_x, e_y, e_z)$ error coordinates
- $R_z(\psi_G)$ rotation matrix

$$\dot{e} = R_z(\chi_G)^T R_z(\chi_A) R_y(\gamma_A) \begin{bmatrix} v_A \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} (1 - e_y\sigma_G)v_G \\ e_x\sigma_G v_G \\ 0 \end{bmatrix}.$$

Constraints for successful rendezvous

Given the coupled UAV-UGV dynamics



Error space frames.

$$e = R_z(\psi_G)^\top (p_A - p_G)$$

$$e_\chi = \chi_A - \chi_G$$

- $e_z \leq 0$ (avoid collision)
- $|e_\chi| \leq \left(\frac{e_x}{\bar{e}_x}\right)^2 + \left(\frac{e_y}{\bar{e}_y}\right)^2 + \left(\frac{e_z}{\bar{e}_z}\right)^2 + \bar{e}_\chi$ (physical docking)

Optimal control problem: trajectory-tracking

We now address the problem of computing rendezvous trajectories by using nonlinear least squares trajectory optimization:

$$\min_{\mathbf{x}(\cdot),\mathbf{u}(\cdot)} \frac{1}{2}\int_0^T \|\mathbf{x}(t)-\mathbf{x}^d(t)\|_Q^2+\|\mathbf{u}(t)-\mathbf{u}^d(t)\|_R^2 dt + \frac{1}{2}\|\mathbf{x}(T)-\mathbf{x}^d(T)\|_{P_1}^2$$

subj. to *dynamics constraints*

state/input constraints

where $(\mathbf{x}^d(\cdot),\mathbf{u}^d(\cdot))$ is a desired curve, $T>0$ is fixed, and Q, R and P_1 are positive definite weighting matrices.

Optimal control problem: trajectory-tracking

$$\min_{\mathbf{x}(\cdot),\mathbf{u}(\cdot)} \frac{1}{2}\int_0^T \|\mathbf{x}(t)-\mathbf{x}^d(t)\|_Q^2+\|\mathbf{u}(t)-\mathbf{u}^d(t)\|_R^2 dt + \frac{1}{2}\|\mathbf{x}(T) - \mathbf{x}^d(T)\|_{P_1}^2$$

subj. to *dynamics constraints*

state/input constraints

Goal: compute the optimal trajectory that is close in the L_2 norm to the desired curve $(\mathbf{x}^d(\cdot), \mathbf{u}^d(\cdot))$ (path and velocity).

Desired trajectory:

- e_x^d, e_y^d, e_z^d, and μ^d are set to zero.
- $v_A^d = v_G^d = v_{A\ min}$.
- γ_A^d and ϕ_A^d are set to zero.
- $u_1^d = T_{eq}(v_A^d)$, u_2^d, u_3^d, u_4^d are set to zero.

Optimal control problem: trajectory-tracking

$$\min_{\mathbf{x}(\cdot),\mathbf{u}(\cdot)} \frac{1}{2}\int_0^T \|\mathbf{x}(t)-\mathbf{x}^d(t)\|_Q^2+\|\mathbf{u}(t)-\mathbf{u}^d(t)\|_R^2 dt + \frac{1}{2}\|\mathbf{x}(T)-\mathbf{x}^d(T)\|_{P_1}^2$$

subj. to *dynamics constraints*

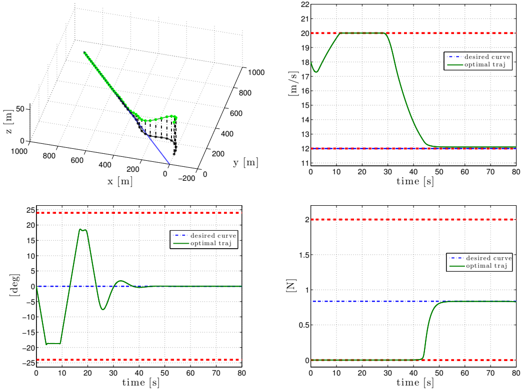
state/input constraints

Remark

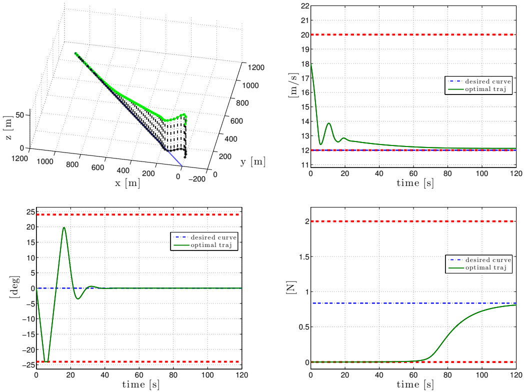
Q and R allow us to tune specific performance features of the UAV. By a suitable choice of the weighting matrices, the trajectory can be highly aggressive or it can be very smooth.

A. Rucco, P. Sujit, A. P. Aguiar, and J. Sousa, "Optimal UAV rendezvous on a UGV," in Proceedings of the AIAA Guidance, Navigation, and Control Conference, 2016

Aggressive trajectory

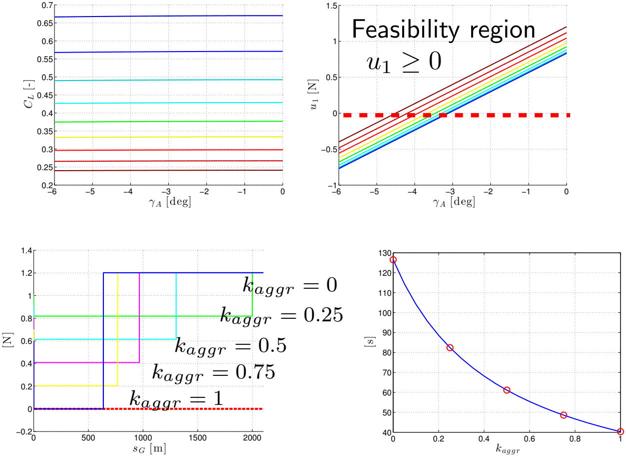


Soft trajectory

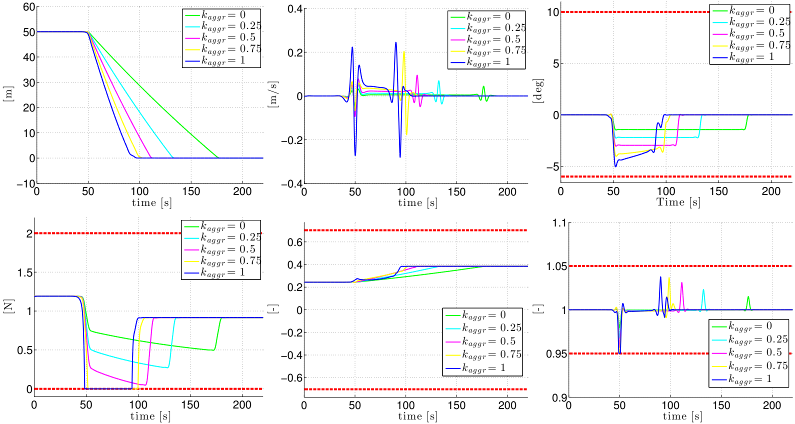


Rendezvous strategy: main features

- aggressiveness index based on the maximum UAV capability
- desired state-input curve based on the decoupled UAV-UGV

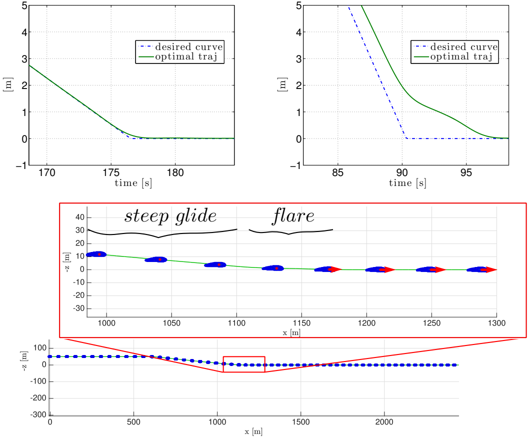


Rendezvous on a straight line path



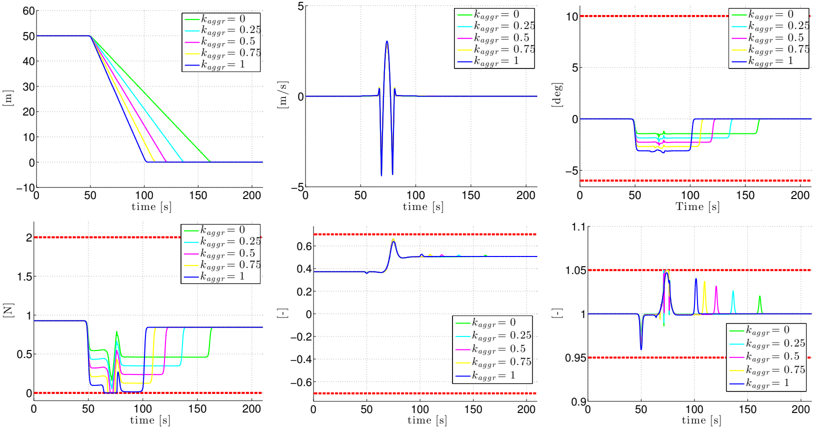
A. Rucco, P.B. Sujit, A.P.Aguiar, J.B.Sousa, F.L.Pereira, "Optimal Rendezvous Trajectory for Unmanned Aerial-Ground Vehicles," IEEE TAES 2017.

Rendezvous strategy on a straight line path



A. Rucco, P.B. Sujit, A.P.Aguiar, J.B.Sousa, F.L.Pereira, "Optimal Rendezvous Trajectory for Unmanned Aerial-Ground Vehicles," IEEE TAES 2017.

Rendezvous strategy on a turn



A. Rucco, P.B. Sujit, A.P.Aguiar, J.B.Sousa, F.L.Pereira, "Optimal Rendezvous Trajectory for Unmanned Aerial-Ground Vehicles," IEEE TAES 2017.

Conclusions

Optimization framework for motion planning of UAVs
in dynamic environments

Optimal control based strategies to enforce
trajectory-tracking or path-following behaviour

Novel approach for generating smooth UAV trajectories
(application to target tracking of marine vehicles)

Strategy which allows to select the type of UAV trajectory
(application to UAV-UGV rendezvous)

Conclusions

Thank you!