Cooperative Motion Planning for Multiple Autonomous Underwater Vehicles

Bahareh Sabetghadam

LARSyS Robotics and Systems in Engineering and Science

DSOR

TÉCNICO LISBOA

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Summary

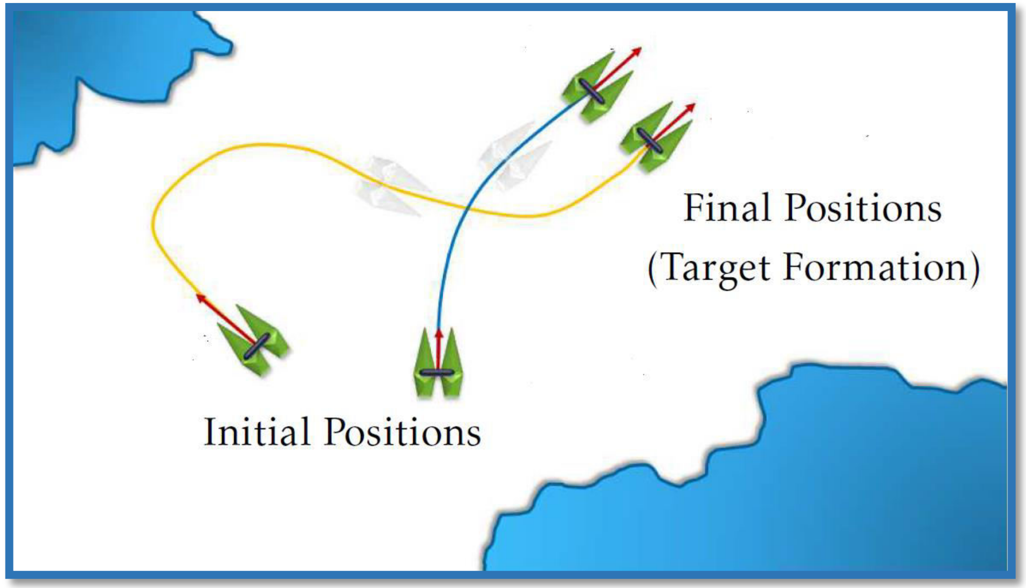
- Examples

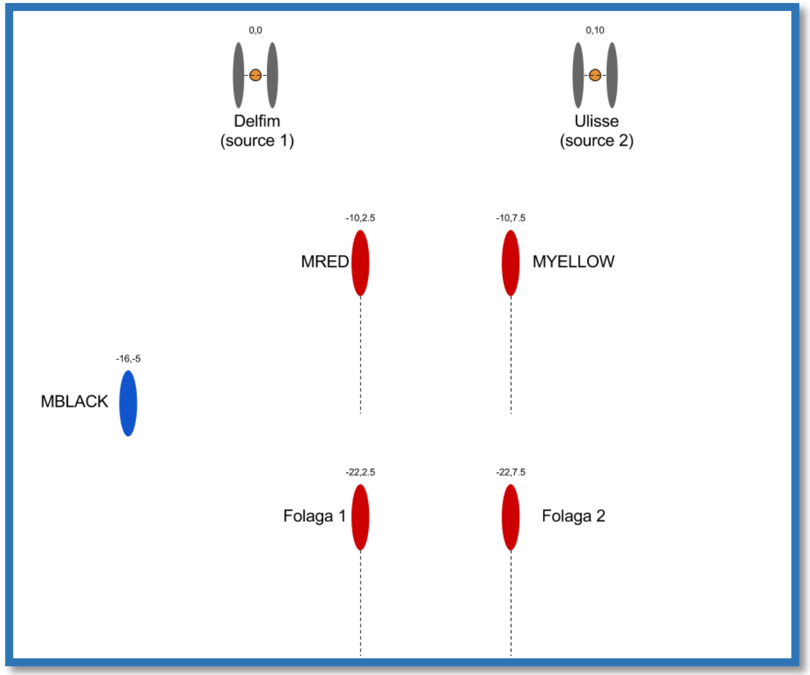
  Go-to-Formation Maneuver

  Range-Based Vehicle Positioning and Target Localization

- Problem Formulation
- Optimal Trajectory generation Methods
- Polynomial-Based Methods

Go-to-Formation Maneuver

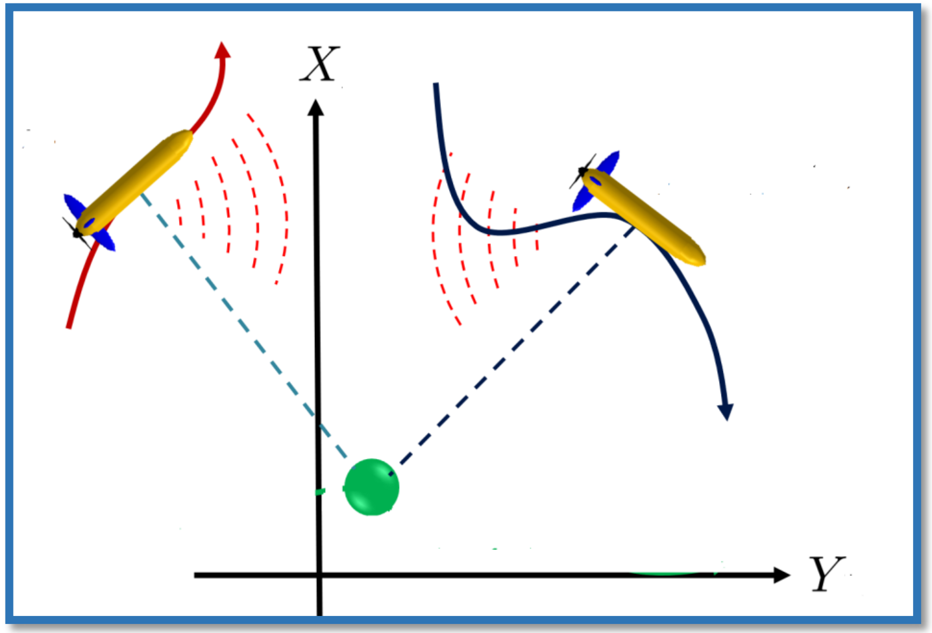




Objectives:

- Driving the vehicles from the initial positions to the target formation
- Ensuring simultaneous arrival at final positions with desired speed and heading

Range-Based Vehicle Positioning and Target Localization



Objectives:

- Obtaining a sequence of range measurements to fixed or moving beacons with known positions
- Maximizing the range-based information available for an accurate estimation of the vehicle position

Trajectory Generation Problem for N vehicles

$$\min_{\substack{P^{[i]} \\ i=1,\dots,N}} J(\cdot)$$

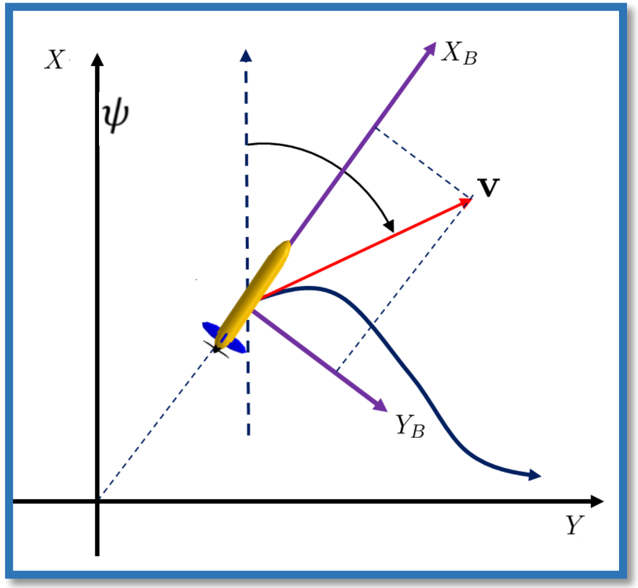
$$\begin{aligned}
\textit{subject to} \;\; & \textit{boundary conditions}, \\
& \textit{vehicle dynamic constraints}, \\
& \textit{temporal or spatial deconfliction constraints}, \\
& \textit{obstacle avoidance constraints}, \\
& \textit{mission specific constraints},
\end{aligned}$$

- $P^{[i]}$ is the i^{th} vehicle trajectory.
- $J(\cdot)$ is the given cost function.

Problem Constraints

The trajectories should satisfy the constraints imposed by the vehicle dynamics,

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{v} \\ \dot{\psi} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} v\cos(\psi) \\ v\sin(\psi) \\ 0 \\ r \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$



- upper/lower bounds on state variables,

$$v_{min} \le v(t) \le v_{max} \qquad \psi_{min} \le \psi(t) \le \psi_{max} \qquad r_{min} \le r(t) \le r_{max}$$

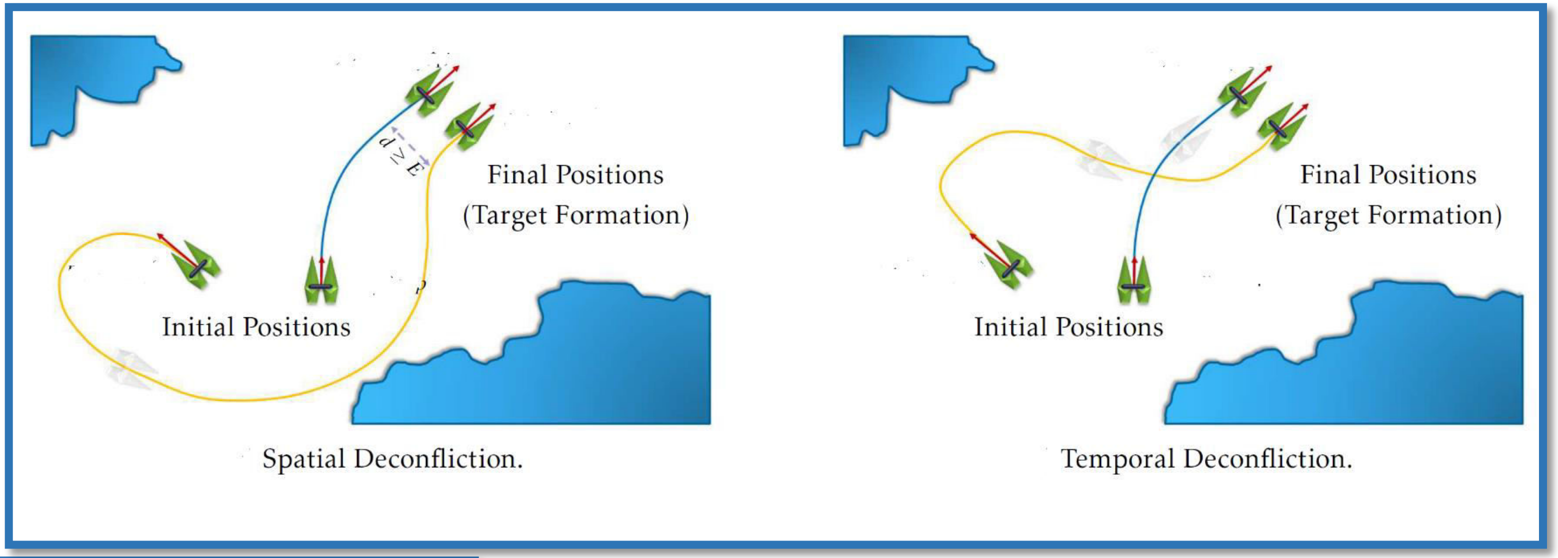
- upper/lower bounds on input variables,

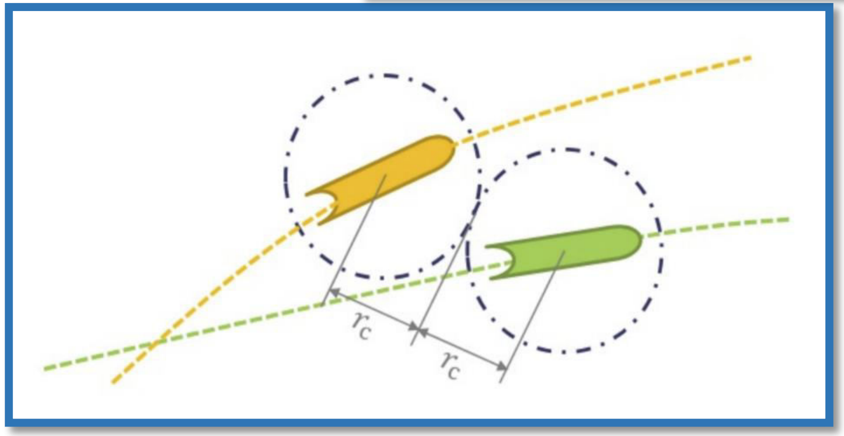
$$u_{1_{min}} \le u_1(t) \le u_{1_{max}} \qquad u_{2_{min}} \le u_2(t) \le u_{2_{max}}$$

- Dynamic constraints are considered either explicitly or in terms of geometrical properties of the path.

Problem Constraints

The trajectories should be collision-free,





$$c_{col}\left(p^{[i]}(t), p^{[j]}(t)\right) = \frac{\left(x^{[i]}(t) - x^{[j]}(t)\right)^2}{(2r_c)^2} + \frac{\left(y^{[i]}(t) - y^{[j]}(t)\right)^2}{(2r_c)^2} - 1 \geq 0$$

Objective Function

The trajectories should minimize (or maximize) a cost function.

- Minimizing travel time or energy consumption

- Maximizing the information available for estimation

Using the **Fisher Information Matrix** to quantify the information content of measurements

FIM Basics

For the random variable θ with probability distribution P_θ,

$$FIM_\theta \stackrel{\text{def}}{=} \mathbb{E}\left[\left(\nabla_\theta \Lambda_\theta\right)\left(\nabla_\theta \Lambda_\theta\right)^{\mathrm{T}}\right]$$

where Λ_θ is the log likelihood function

$$\Lambda_\theta \stackrel{\text{def}}{=} \ln p_\theta(X)$$

Trajectory Generation Problem as an OCP

$$
\begin{aligned}
\min \quad & \int_0^{t_f} \sum_{i=1}^{N_v} L_i(\boldsymbol{x}^{[i]}(\tau), \boldsymbol{u}^{[i]}(\tau))\, d\tau + \sum_{i=1}^{N_v} E_i(\boldsymbol{x}^{[i]}(t_f)) \\
s.t. \quad & \dot{\boldsymbol{x}}^{[i]}(t) = f_i\left(\boldsymbol{x}^{[i]}, \boldsymbol{u}^{[i]}\right) \\
& \boldsymbol{x}^{[i]}(0) = \boldsymbol{x}_0^{[i]} \\
& \boldsymbol{x}^{[i]}\left(t_f\right) = \boldsymbol{x}_f^{[i]} \\
& c_{col}\left(\boldsymbol{x}^{[i]}(t), \boldsymbol{x}^{[j]}(t)\right) \geq 0 \\
& \underline{\boldsymbol{x}}^{[i]} \leq \boldsymbol{x}^{[i]}(t) \leq \overline{\boldsymbol{x}}^{[i]} \\
& \underline{\boldsymbol{u}}^{[i]} \leq \boldsymbol{u}^{[i]}(t) \leq \overline{\boldsymbol{u}}^{[i]}
\end{aligned}
$$

- The constraints should be satisfied for all $t \in [0, t_f]$ with $i, j \in \{1, \dots, N_v\}$.

Optimal Trajectory Generation Methods and Tools

- PRojection Operator based Newton method for Trajectory Optimization (PRONTO)

- Direct Optimal Control Methods,

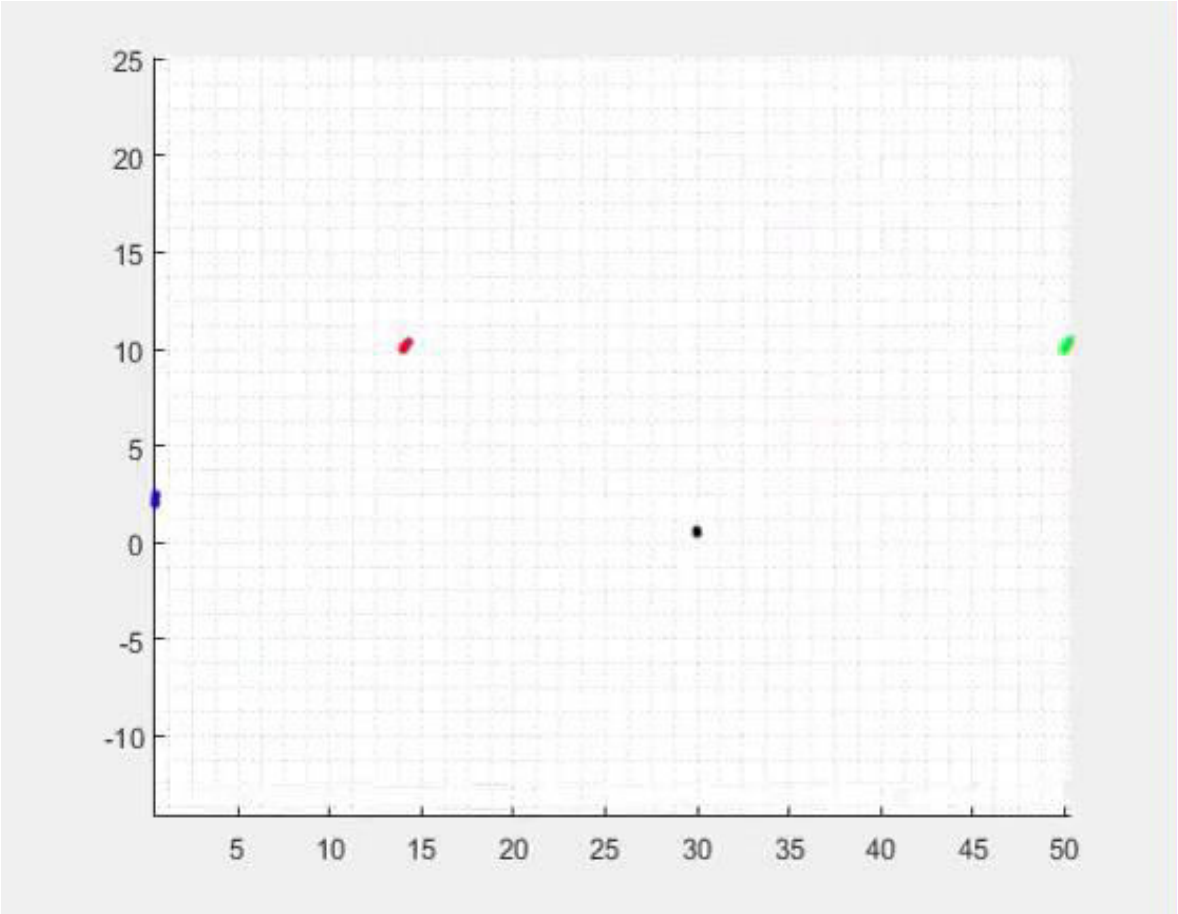
  - Collocation (Pseudo-spectral methods)

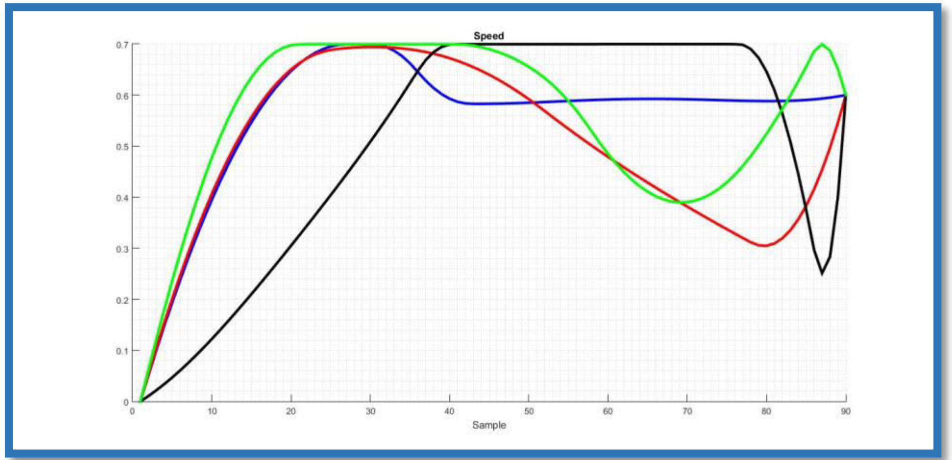
    DIDO

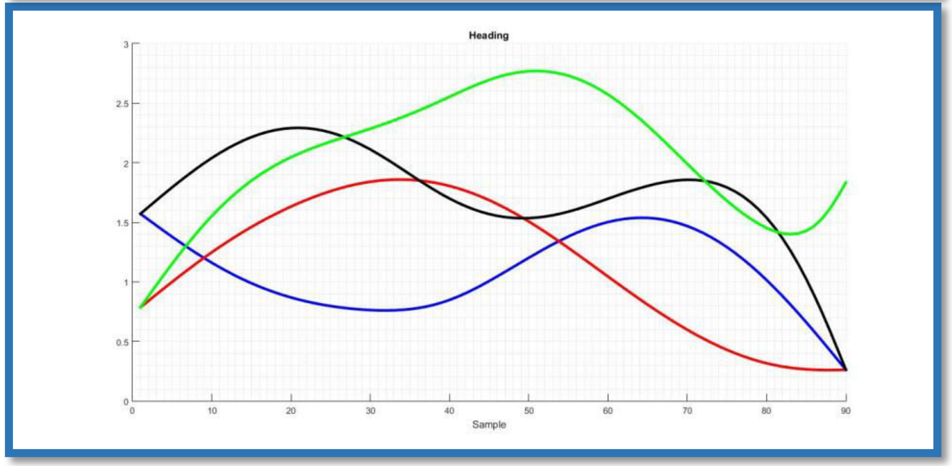
  - Multiple Shooting

    NLP solvers (**FORCES Pro**, …)

Simulation Results

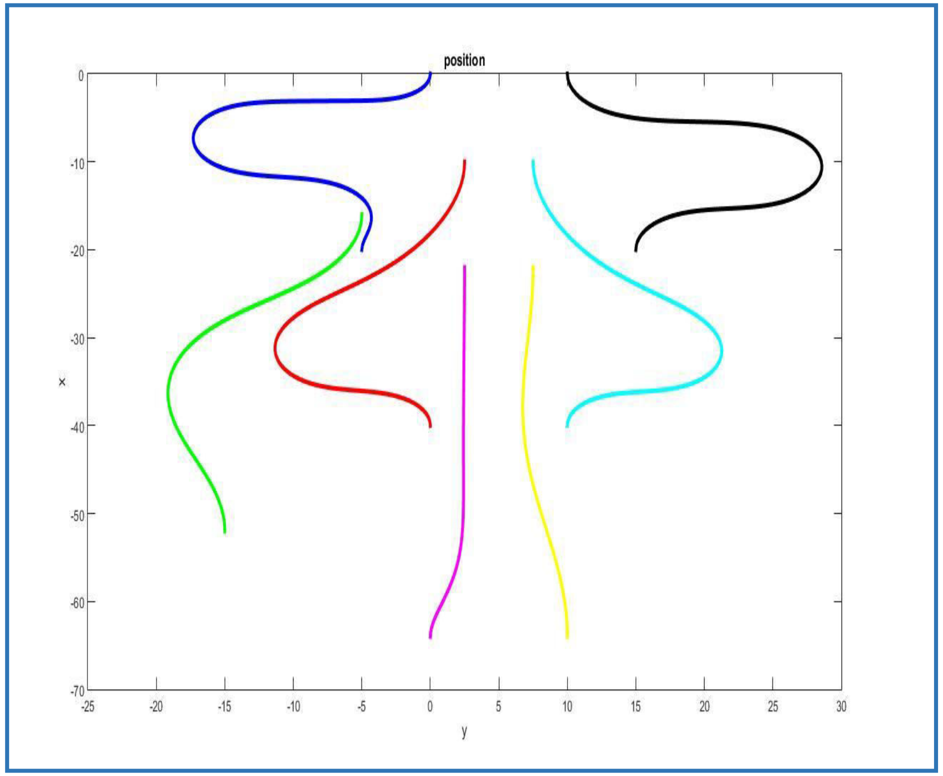


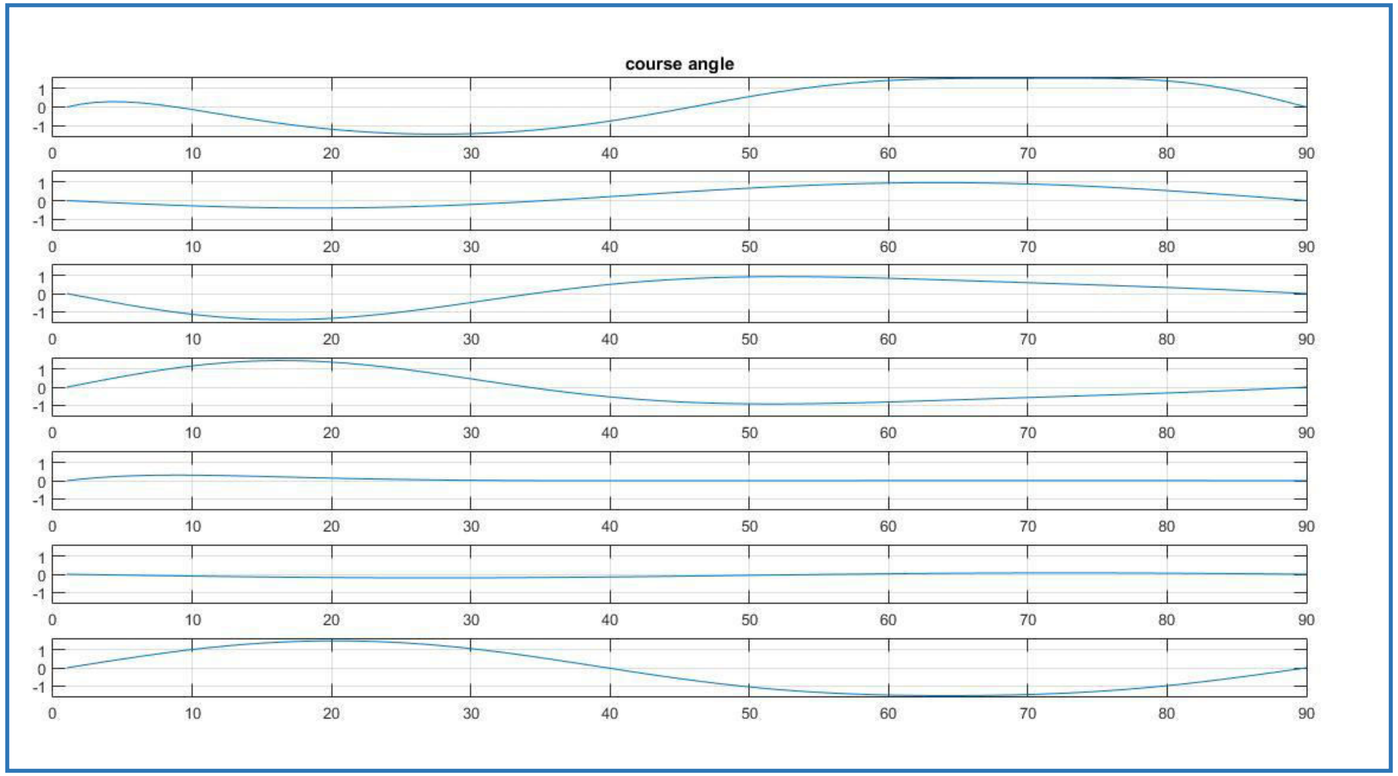




- Temporally deconflicted trajectories for 4 vehicles minimizing time and energy
- Problem solved in **6067 milliseconds** using FORCES Pro interior-point solver

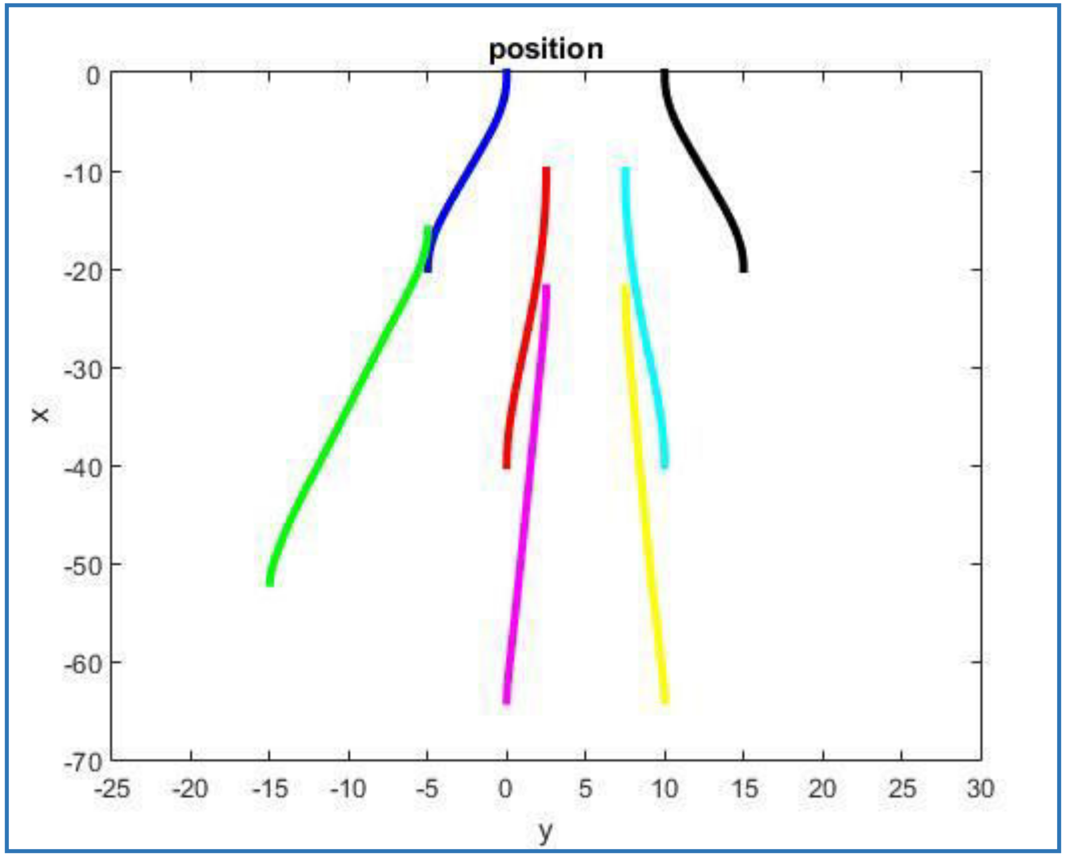
Simulation Results

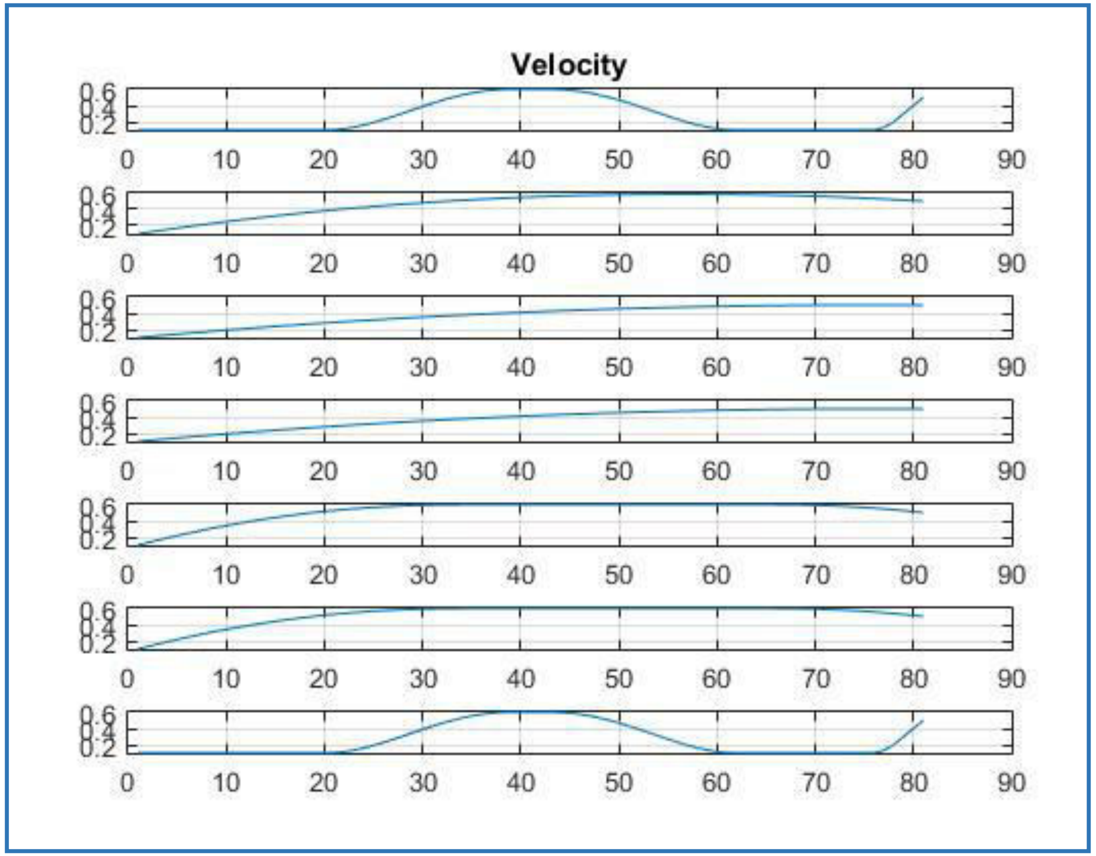




- Spatially deconflicted trajectories for 7 vehicles with constant speed
- Problem solved in **39 seconds** on a desktop computer with 2.60 GHz i7-4510U CPU and 6.00 GB RAM

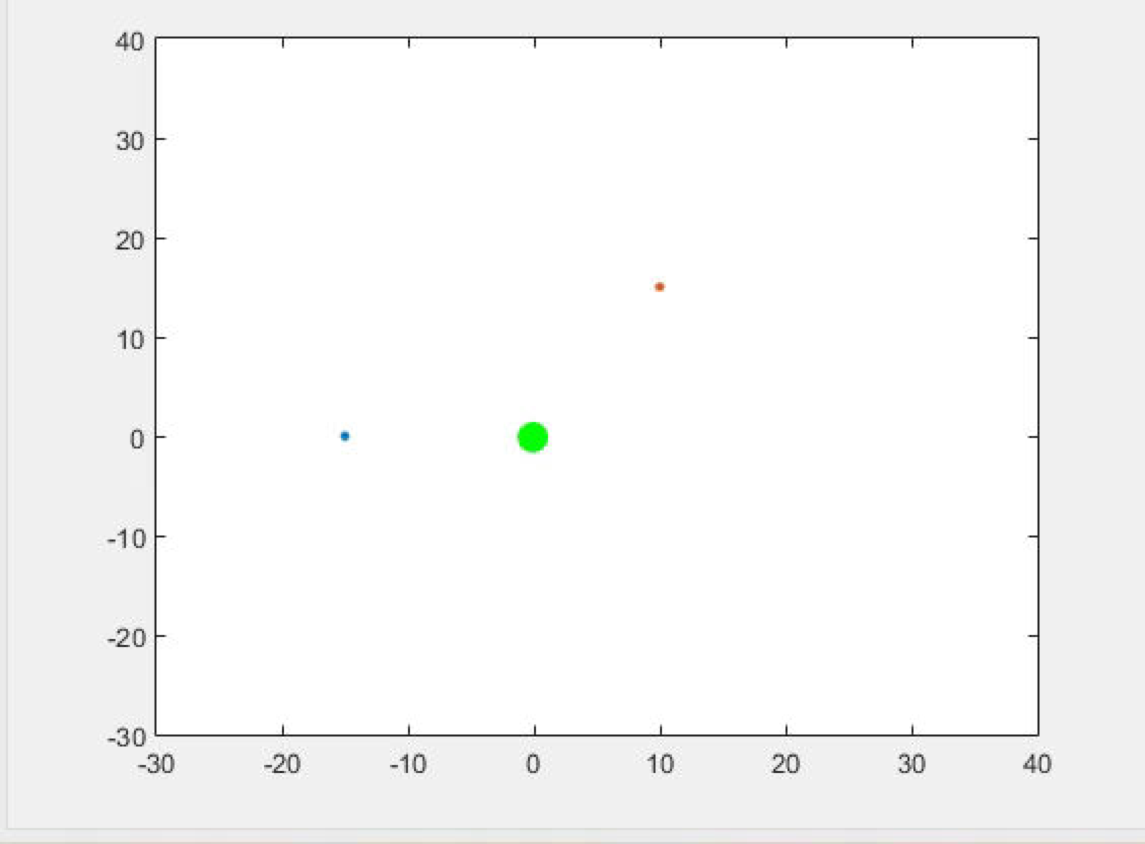
Simulation Results





- Same problem with variable speed bounded between 0.2m/s and 0.6m/s

Simulation Results



- Maximizing the overall FIM for two vehicles one beacon problem
- Problem solved in **5046 milliseconds**

$$\det\left(\bigoplus_{i\in\mathbb{I}_p} FIM_{\mathbf{U}_i}^{[i]}(\boldsymbol{\theta}_i)\right) = \prod_{i\in\mathbb{I}_p}\det(FIM_{\mathbf{U}_i}^{[i]}(\boldsymbol{\theta}_i))$$

Polynomial-based Path Planning

- Using polynomial trajectories to describe independent evolution of flat outputs.

- In flat systems, all states and inputs can be expressed in terms of flat output and its derivatives.

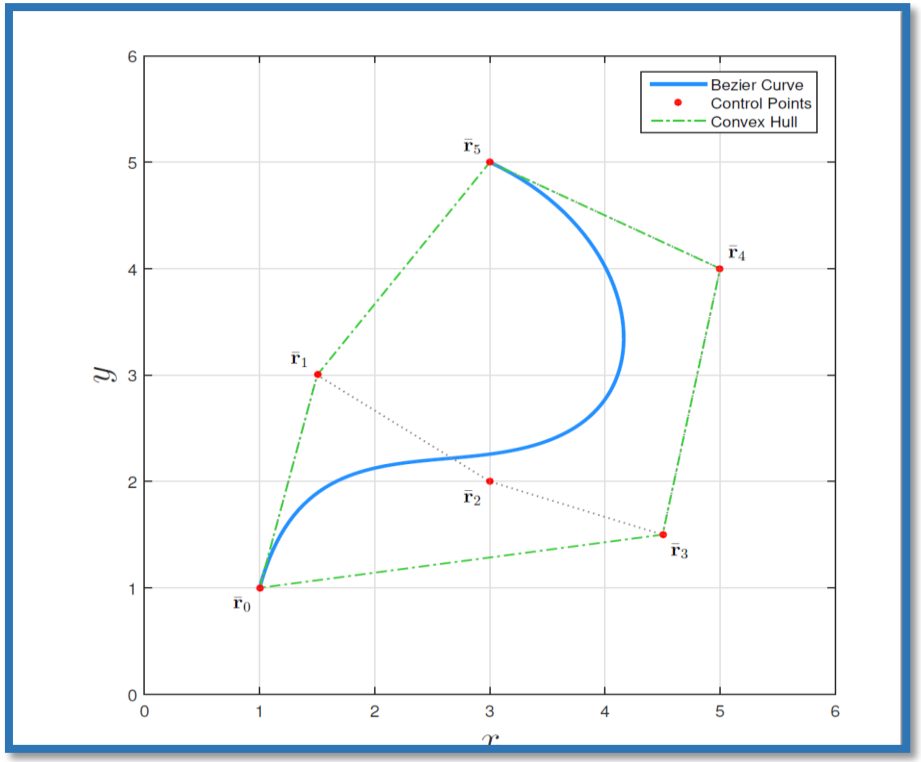
- Exploiting the mathematical and geometric properties of particular classes of curves,

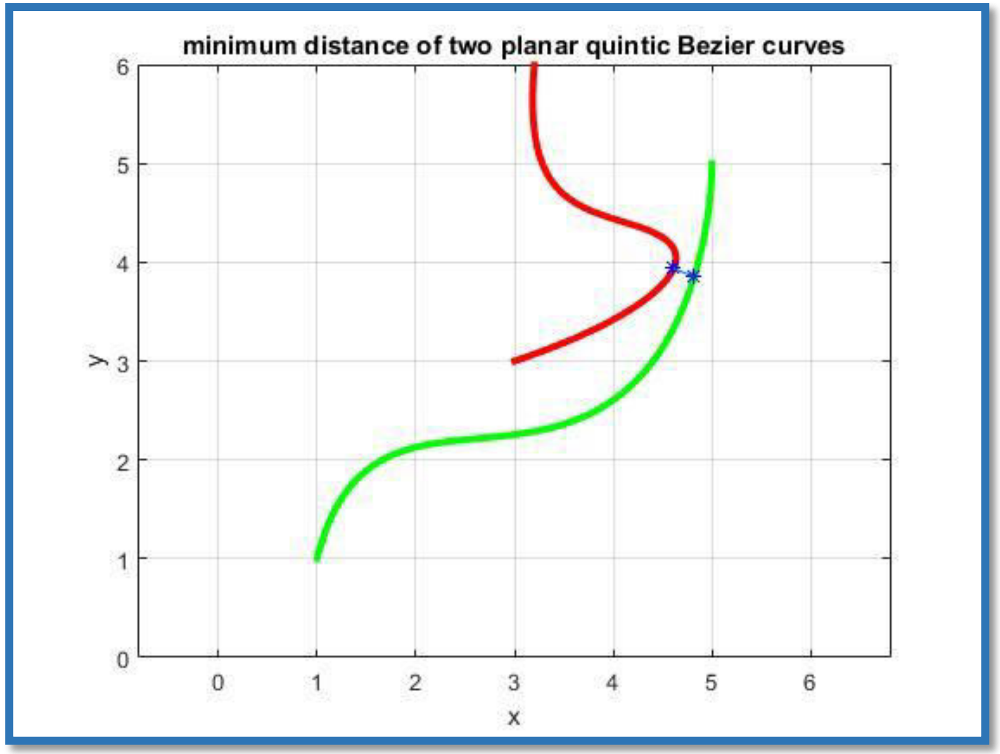
  - **B-splines curves**
  - **Bézier curves**

$$\mathbf{r}(\zeta) = \sum_{k=0}^{n} \bar{\mathbf{r}}_k \, b_k^n(\zeta)\,,$$

$$b_k^n(\zeta) = \binom{n}{k} (1-\zeta)^{n-k} \zeta^k\,, \qquad \zeta \in [0,1]$$

Properties of Bezier Curve





- The curve is inside the convex hull defined by the control points.
- The minimum distance can be computed efficiently using De Casteljau’s algorithm.

Polynomial-based Path Planning

- Spatial path

  Polynomial in dimensionless parameter τ,

$$P^i(\tau) = \begin{bmatrix} P_1^i(\tau) \\ P_2^i(\tau) \end{bmatrix} = \begin{bmatrix} \sum_{k=0}^{n} p_{2,k}^i \tau^k \\ \sum_{k=0}^{n} p_{1,k}^i \tau^k \end{bmatrix} \qquad \tau \in [0, \tau_f^i]$$

- Timing law

  Non-negative function of time,

$$\Theta^i(t) \coloneqq \frac{d\tau}{dt} = \sum_{k=0}^{n_\Theta} \theta_k^i t^k \qquad t \in [0, t_f]$$

Problem Constraints

- Expressing constraints in terms of spatial path and timing law

Dynamic Constraints $\longrightarrow$

$$v_{min}^{i} \leq \sigma^{i}(\tau^{i}(t))\,\Theta^{i}(t) \leq v_{max}^{i}$$

$$\sin(\psi_{min}) \leq \sin\big(\psi(\tau)\big) \leq \sin(\psi_{max})$$

$$a(t) := \frac{dv}{dt} = \dot{\Theta}(t)\|P'(\tau)\| + \Theta^2(t)\sigma'(\tau)$$

$$\dot{\psi}(t) = \frac{(\|P'(\tau)\|P_2''(\tau) - \sigma'(\tau)P_2'(\tau))\Theta(t)}{\|P'(\tau)\|P_1'(\tau)}$$

- where

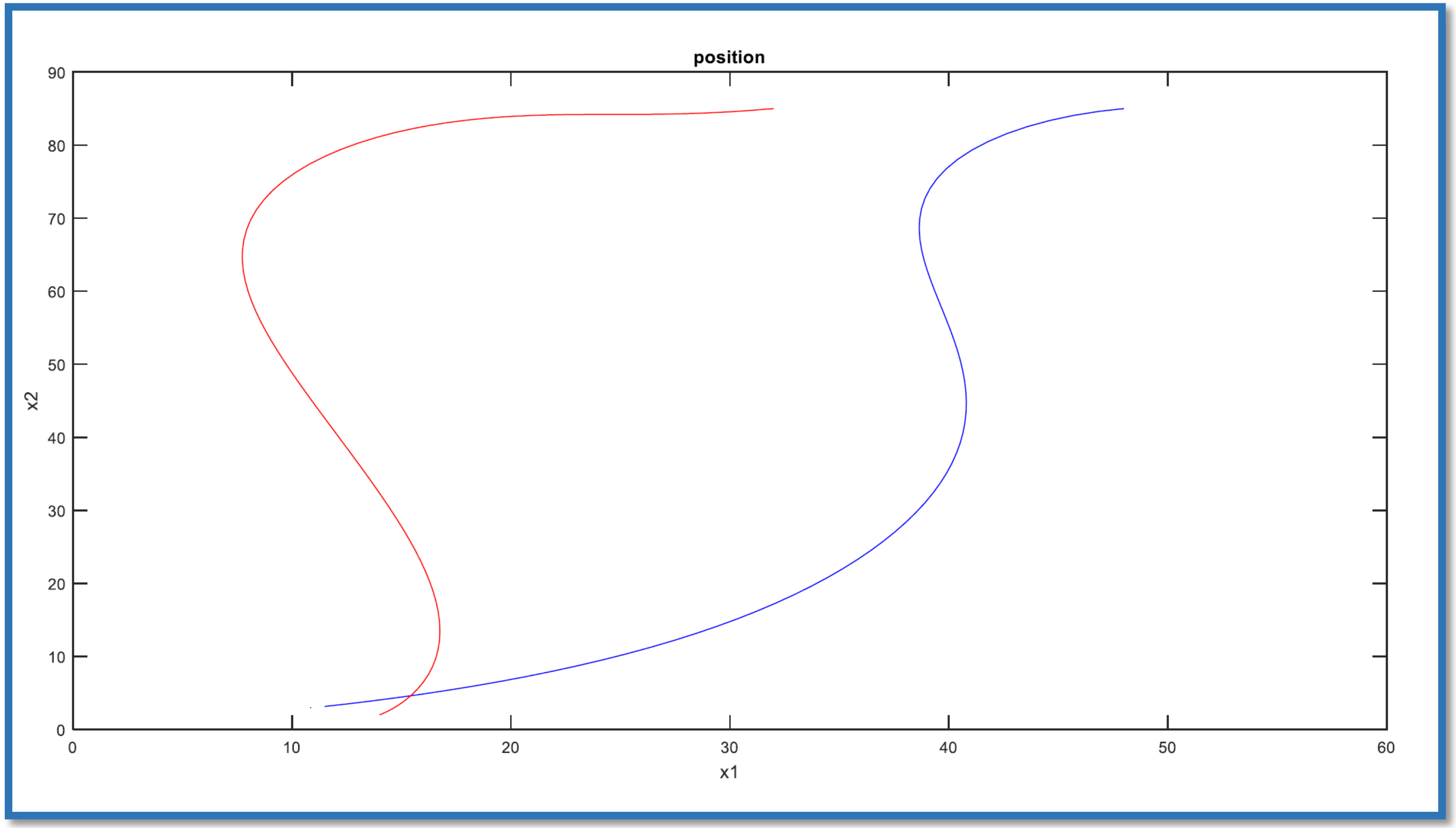
$$\sin\big(\psi(\tau)\big) = \frac{P_2'(\tau)}{\|P'(\tau)\|} \qquad \sigma'(\tau) = \frac{\partial\sigma(\tau)}{\partial\tau} = \frac{P'(\tau).P''(\tau)}{\|P'(\tau)\|} \qquad \sigma^{i}(\tau) = \|P^{i\prime}(\tau)\|$$

- Temporal and spatial deconfliction constraints,

$$\left\|P^{i}(\tau^{i}) - P^{j}(\tau^{j})\right\|^2 \geq d_{ij} \quad \text{for} \quad i,j \in \{1,\dots,N_v\} \text{ and } \forall\, \tau^{i},\tau^{j} \in [0,\tau_f]$$

$$\left\|P^{i}(t) - P^{j}(t)\right\|^2 \geq d_{ij} \quad \text{for} \quad i,j \in \{1,\dots,N_v\} \text{ and } \forall\, t \in [0,t_f]$$

Simulation Results



- Temporally de-conflicted trajectories for two vehicles using quantic Bezier Curves
- Problem solved in **2032 milliseconds**

Thank you!