Cooperative Motion Planning for Multiple Autonomous Underwater Vehicles

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Summary

- Examples
  
  Go-to-Formation Maneuver
  
  Range-Based Vehicle Positioning and Target Localization

- Problem Formulation

- Optimal Trajectory generation Methods

- Polynomial-Based Methods
Objectives:

- Driving the vehicles from the initial positions to the target formation
- Ensuring simultaneous arrival at final positions with desired speed and heading
Objectives:

- Obtaining a sequence of range measurements to fixed or moving beacons with known positions
- Maximizing the range-based information available for an accurate estimation of the vehicle position
Trajectory Generation Problem for N vehicles

\[
\begin{align*}
\min_{p^{[i]}} & \quad J(\cdot) \\
\text{s.t.} & \quad i=1,...,N
\end{align*}
\]

subject to boundary conditions,

- vehicle dynamic constraints,
- temporal or spatial deconfliction constraints,
- obstacle avoidance constraints,
- mission specific constraints,

- \( P^{[i]} \) is the \( i^{th} \) vehicle trajectory.
- \( J(\cdot) \) is the given cost function.
Problem Constraints

The trajectories should satisfy the constraints imposed by the vehicle dynamics,

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
v \cos(\psi) \\
v \sin(\psi) \\
r
\end{bmatrix} +
\begin{bmatrix}
0 & 0 \\
0 & 0 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix}
\]

- upper/lower bounds on state variables,

\[
v_{\text{min}} \leq v(t) \leq v_{\text{max}} \quad \psi_{\text{min}} \leq \psi(t) \leq \psi_{\text{max}} \quad r_{\text{min}} \leq r(t) \leq r_{\text{max}}
\]

- upper/lower bounds on input variables,

\[
u_{1\text{min}} \leq u_1(t) \leq u_{1\text{max}} \quad u_{2\text{min}} \leq u_2(t) \leq u_{2\text{max}}
\]

- Dynamic constraints are considered either explicitly or in terms of geometrical properties of the path.
Problem Constraints

The trajectories should be collision-free,

\[
c_{col}(p^{[i]}(t), p^{[j]}(t)) = \frac{(x^{[i]}(t) - x^{[j]}(t))^2}{(2r_c)^2} + \frac{(y^{[i]}(t) - y^{[j]}(t))^2}{(2r_c)^2} - 1 \geq 0
\]
Objective Function

The trajectories should minimize (or maximize) a cost function.

- Minimizing travel time or energy consumption
- Maximizing the information available for estimation

Using the Fisher Information Matrix to quantify the information content of measurements

**FIM Basics**

For the random variable $\theta$ with probability distribution $P_\theta$, $FIM_\theta \overset{\text{def}}{=} \mathbb{E} \left[ (\nabla_\theta \Lambda_\theta) (\nabla_\theta \Lambda_\theta)^T \right]$

where $\Lambda_\theta$ is the log likelihood function

$\Lambda_\theta \overset{\text{def}}{=} \ln p_\theta(X)$
Trajectory Generation Problem as an OCP

\[
\begin{align*}
\min \quad & \int_0^{t_f} \sum_{i=1}^{N_v} L_i(x^{[i]}(\tau), u^{[i]}(\tau)) \, d\tau + \sum_{i=1}^{N_v} E_i(x^{[i]}(t_f)) \\
\text{s.t.} \quad & \dot{x}^{[i]}(t) = f_i(x^{[i]}, u^{[i]}) \\
& x^{[i]}(0) = x_0^{[i]} \\
& x^{[i]}(t_f) = x_f^{[i]} \\
& c_{col}(x^{[i]}(t), x^{[j]}(t)) \geq 0 \\
& \underline{x}^{[i]} \leq x^{[i]}(t) \leq \bar{x}^{[i]} \\
& \underline{u}^{[i]} \leq u^{[i]}(t) \leq \bar{u}^{[i]} 
\end{align*}
\]

- The constraints should be satisfied for all \( t \in [0, t_f] \) with \( i, j \in \{1, ..., N_v\} \).
Optimal Trajectory Generation Methods and Tools

- PRojection Operator based Newton method for Trajectory Optimization (PRONTO)

- Direct Optimal Control Methods,
  - Collocation (Pseudo-spectral methods)
    - DIDO
  - Multiple Shooting
    - NLP solvers (FORCES Pro, ...)

Simulation Results

- Temporally deconflicted trajectories for 4 vehicles minimizing time and energy
- Problem solved in 6067 milliseconds using FORCES Pro interior-point solver
Simulation Results

- Spatially deconflicted trajectories for 7 vehicles with constant speed
- Problem solved in **39 seconds** on a desktop computer with 2.60 GHz i7-4510U CPU and 6.00 GB RAM
Simulation Results

- Same problem with variable speed bounded between 0.2m/s and 0.6m/s
Simulation Results

• Maximizing the overall FIM for two vehicles one beacon problem
• Problem solved in **5046 milliseconds**
Polynomial-based Path Planning

• Using polynomial trajectories to describe independent evolution of flat outputs.

• In flat systems, all states and inputs can be expressed in terms of flat output and its derivatives.

• Exploiting the mathematical and geometric properties of particular classes of curves,
  - B-splines curves
  - Bézier curves

\[
\mathbf{r}(\zeta) = \sum_{k=0}^{n} \mathbf{r}_k b_k^n(\zeta),
\]

\[
b_k^n(\zeta) = \binom{n}{k} (1 - \zeta)^{n-k} \zeta^k, \quad \zeta \in [0, 1]
\]
Properties of Bezier Curve

- The curve is inside the convex hull defined by the control points.
- The minimum distance can be computed efficiently using De Casteljau’s algorithm.
 Polynomial-based Path Planning

• Spatial path
  Polynomial in dimensionless parameter $\tau$,

$$P^i(\tau) = \begin{bmatrix} P_1^i(\tau) \\ P_2^i(\tau) \end{bmatrix} = \begin{bmatrix} \sum_{k=0}^{n} p_{2,k}^{i} \tau^k \\ \sum_{k=0}^{n} p_{1,k}^{i} \tau^k \end{bmatrix} \quad \tau \in [0, \tau_f^i]$$

• Timing law
  Non-negative function of time,

$$\Theta^i(t) := \frac{d\tau}{dt} = \sum_{k=0}^{n_{\Theta}} \theta_k^{i} t^k \quad t \in [0, t_f]$$
Problem Constraints

• Expressing constraints in terms of spatial path and timing law

\[ v_{min}^i(t) \leq \sigma^i(\tau^i(t)) \theta^i(t) \leq v_{max}^i \]
\[ \sin(\psi_{min}) \leq \sin(\psi(t)) \leq \sin(\psi_{max}) \]
\[ a(t) := \frac{dv}{dt} = \dot{\theta}(t)\|P'(\tau)\| + \Theta^2(t)\sigma'(\tau) \]
\[ \dot{\psi}(t) = \frac{(\|P'(\tau)\|P''(\tau) - \sigma'(\tau)P'_2(\tau))\Theta(t)}{\|P'(\tau)\|P'_1(\tau)} \]

Dynamic Constraints

• where

\[ \sin(\psi(\tau)) = \frac{p'_1(\tau)}{\|P'(\tau)\|} \]
\[ \sigma'(\tau) = \frac{\partial \sigma(\tau)}{\partial \tau} = \frac{P'(\tau)\cdot P''(\tau)}{\|P'(\tau)\|} \]
\[ \sigma^i(\tau) = \|P''(\tau)\| \]

• Temporal and spatial deconfliction constraints,

\[ \|P^i(\tau^i) - P^j(\tau^j)\|^2 \geq d_{ij} \quad \text{for} \quad i, j \in \{1, ..., N_v\} \quad \text{and} \quad \forall \tau^i, \tau^j \in [0, \tau_f] \]
\[ \|P^i(t) - P^j(t)\|^2 \geq d_{ij} \quad \text{for} \quad i, j \in \{1, ..., N_v\} \quad \text{and} \quad \forall t \in [0, \tau_f] \]
Simulation Results

- Temporally de-conflicted trajectories for two vehicles using quantic Bezier Curves
- Problem solved in **2032 milliseconds**
Thank you!